Understanding market value margins in options prices

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At the heart of the emerging Solvency II regulations is the idea that both assets and liabilities should be valued on a consistent basis using market values where they are available. Where markets do not exist – where exposures are deemed to be non-hedgeable – an alternative approach is required. Until recently most practitioners would have expected to adopt one of two approaches. Firstly, a mark-to-model approach for non-hedgeable market risk which recognises and implicitly adjusts for the hard-to-measure cost of market risk capital. Secondly, a 'cost-of-capital' (CoC) approach for dealing with non-hedgeable non-market risk. Whilst these approaches make fundamentally different assumptions about the economic characteristics of the underlying assets or liabilities, regulators appear to be seriously considering collapsing them into a single CoC approach. The outright rejection of mark-to-model would have some major implications for valuation, our own extrapolation methodologies and the viability of hedging strategies for insurance firms.

The purpose of this note is to compare and contrast a market-based approach to estimating the costs of risk capital that are embedded in asset prices with the cost-of-capital method. Our aim is to highlight the significant challenge in discovering appropriate discount rates in the presence of market risk using a cost-of-capital approach and contrast this with the conventional mark-to-model method using a simple example using option values.

Background

Economic exposures on the insurance balance sheet are conventionally classified into four buckets which allow for whether a cash flow is linked to financial asset prices and whether it is hedgeable i.e. whether a market exists for such a cash flow.

Financial	Hedgeable
Financial	Non-Hedgeable
Non-Financial	Hedgeable
Non-Financial	Non-Hedgeable

Hedgeable cash flows are straightforward to value. We can simply use a market price from a *deep, liquid and transparent* market. As an aside, you should note that the definition of a *deep, liquid and transparent* market has been hardened up in the recent CEIOPS consultation papers #39 and #41 so that only a narrow range of financial markets may qualify. Unhedgeable cash flows are more problematic. For non-financial cash flows such as insurance risks, firms will calculate a 'market value margin' (MVM) in addition to the present value of future liability cash flows. The MVM is an additional allowance for the cost of holding the capital required to meet the liability with a specified level of confidence. The capital is assumed to be equivalent to the Solvency II SCR so the confidence level is set at 99.5%.

A comprehensive explanation of the approach is set out in two papers published by the Chief Risk Officer Forum in 2006 and 2008[1]. Since non-hedgeable non-market risk is assumed to be diversifiable, the CROs recommended a risk discount rate of between 2.5% and 4% pa. Note that this rate is intended to capture the ***frictional*** costs of holding shareholder capital i.e. if a firm invests shareholder capital in a LIBOR fund, it costs the shareholder 2.5%-4% more to do this via an insurance company than to do it directly. These frictional costs are primarily explained as double taxation effects, bankruptcy and agency costs. This is not a required return for bearing market risk exposures and therefore unambiguously was not intended for pricing market risk-contingent cash flows. For Solvency II, CEIOPS have proposed a somewhat higher rate of 6% pa.

Interestingly, it is clear that the authors of the CRO papers expect firms to deal with non-hedgeable ***market*** risk using mark-to-model extrapolation methods. However, the logic of this approach may have been so blindingly obvious to the authors you might easily argue that it did not get the prominence it deserved. The latest CEIOPS thinking now appears to apply the CoC approach in a way we doubt the original authors intended. Let us now contrast the approaches by examining a market price within the CoC framework.

European insurance regulators think of option values in two ways – each intending to capture the actual or hypothetical costs of transfer to a third party – as follows:

1. A market-consistent, fair value calibrated to a market quotation for an option price
2. The sum of the present value of a 'best-estimate' expectation (discounted at the risk-free rate) and an additional market-value margin (MVM) to reflect the additional capital costs likely to be demanded by a counterparty to accept the risk.

This idea has most recently appeared as part of Solvency II discussion where CEIOPS's Consultation Paper 42 (CP42) proposes that Technical Provisions for non-hedgeable cash flows are calculated as the sum of a best estimate value and a risk margin[2]. The best estimate value will be calculated using 'realistic', real-world assumptions for probability.

The remainder of this note is organized in two parts. In the first part we view a standard option price (for which we have a market quote) within this framework to understand the two separate roles performed by the MVM and some lessons for how one might develop fair, model-based 'pseudo' prices for complex or long-maturity assets where no market exists. In the second part we will compare this option value with a best-estimate expectation and Solvency II risk margin using the CoC approach.

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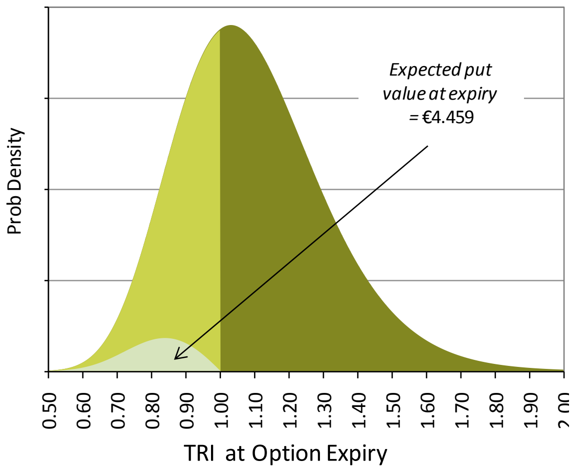
[1] *"A market cost of capital approach to market value margins", CRO Forum, March 2006. "Market Value of Liabilities for Insurance Firms: Implementing elements for Solvency II", CRO Forum, July 2008.*

[2] *CEIOPS-CP 42: Consultation Paper on the Draft L2 Advice on TP – Risk Margin, paragraphs 3.40-3.47, July 2009.*

Market Value Margins within a standard option pricing framework

Chart 1: Total return index distribution under realistic probabilities & put option payoff



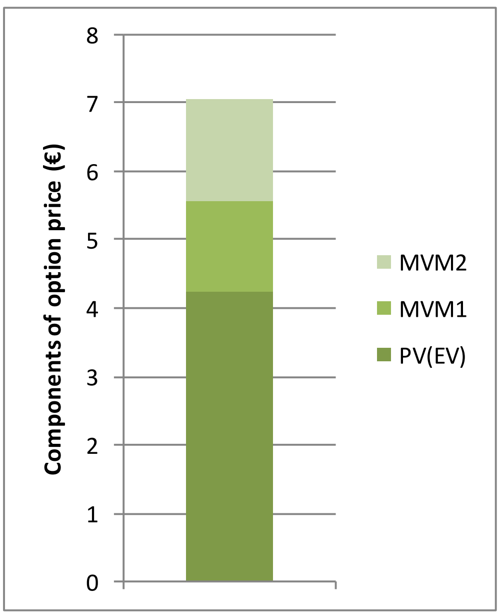
Consider a basic example. Suppose an insurer sells a product which promises a total return on a €100 investment in line with an equity index over a 1-year period with a simple guarantee fixed also at €100. The product (ignoring expenses, taxes etc.) can be replicated with a €100 investment in the underlying equity index and an at-the-money 1-year put option on the total return index. The value of this portfolio tells us the fair value of the liability (irrespective of how the insurer chooses to invest the policyholder's premium).

Let us suppose that the 1-year at-the-money put option sells at a price of €7.081 and the 1-year interest rate is 5% compounded continuously. This implies a volatility input into the standard *Black-Scholes* option pricing formula of 24% pa. Further, let us suppose that we know, *for certain*, the distribution for the end-of-year total return index. It is log normally distributed with a standard deviation of 20% pa and an arithmetic mean of 4% pa (compounding continuously) in excess of the risk-free rate of interest. These assumptions are broadly in line with observed long-term equity risk premia and the relationship between option-implied and delivered equity volatilities[3]. In reality we cannot observe the 'true' distribution and it certainly isn't this convenient shape, but neither matters for our purposes here.

You should note that, if the world really did conform to the assumptions used in the *Black-Scholes* (BS) model – costless markets and a 'diffusion' process for prices, this apparent difference between real-world expectations and option prices would not exist. In a hypothetical BS world the option is an arbitrage price and its payoff can be exactly replicated by holding and continuously adjusting a portfolio of equities and risk-free bonds. In the real world, the costs of trading and jumps in market prices and their volatility means the model is only an approximation and the costs of replication (including capital costs) will be higher. Note that we could try to find the 'true' option pricing model which would give a price of €7.081 with the true end-of-period distribution and true underlying process for price changes. In practice, option traders often find it far more convenient to choose a simple model and adjust the inputs to compensate for the model's 'errors'.

What insights can this simple example offer into the approaches outlined above?

Chart 2: Components of put option value



Firstly, we can calculate[4] the expected payoff from the option which turns out to be €4.459. In other words, on average, given our assumptions, the put option will lose 37% of its initial value over the year. The present value of this cash flow using the 5% risk-free rate is €4.242. This implies a MVM of €2.839 (€7.081 minus €4.242) which is an addition of more than 60% to the discounted expected cash flow.

It is worth analysing this MVM in more detail. Although things are not quite so simple in practice you might like to think of the MVM being comprised of two quite distinct parts (which can be roughly quantified):

1. A component of option value that results from using the 'correct' discount rate instead of the risk-free rate. The index put option has a highly negatively-geared exposure to the underlying equity index. Since the risky equity index is expected to earn a positive risk premium, the negatively geared asset should earn an expected return reflecting its negative market exposure. In this case, in a *Black-Scholes* (BS) world, with volatility of 20% and a risk premium of 4%, this 'fair' expected return would be -20% pa. It is striking that for a relatively simple instrument, because the option's 'beta' is -5.2, the correct BS discount rate is very

[3] *See our real-world equity calibration report: "A comparison of realised and expected volatility", April 2009.*

[4] *In this case the Black-Scholes formula can be used to calculate the payoff on the put option weighted by the real-world probabilities under the log-normal distribution.*

different to the risk-free rate (*very* approximately 5% + (-5.2) * 4%)[5]. Using this adjusted discount rate and the realistic assumptions produces a *Black-Scholes* price of €5.574. This is €1.332 higher than the discounted expected value (let's call this difference MVM1). In other words, if markets really did conform to the assumptions of BS, this is the price at which the option would sell.

2. A second component of the option price arises here because we have assumed a higher volatility is implied in option prices. This is a simple and convenient way of capturing the additional hedging and capital costs of traders in manufacturing and hedging such an option. As we already know, the option sells for €7.081. The additional element (compared to the valuation assuming realistic volatility of 20%) is €1.507. We will label this MVM2.

Market Value Margins using a cost of capital approach

Let us now compare the option-theoretic calculations with the approach to Market Value Margins outlined in the consultation paper. In CP42 CEIOPS introduces the concept of '***unavoidable market risk***' and describes a calculation for how it might be valued. The Solvency II Technical Provision is the sum of a best estimate and a risk margin.

Article 76 of the Solvency II directive states that: *'the best estimate shall correspond to the probability-weighted average of future cash-flows, taking account of the time value of money (expected present value of future cash-flows), using the relevant risk-free interest rate term structure'*.

The calculation of the risk margin is described in CP42 as a Cost of Capital charge (provisionally set at 6% pa) on the required capital. The required capital is calculated at a 99.5% confidence level over 1 year. Let us assume that the market price we have obtained for the 1-year option has been obtained from a market that does ***not*** meet the (stringent) *"deep, liquid and transparent"* market requirements described in consultation papers #39 and #41. To calculate the Technical Provision we therefore are required to calculate the best estimate and the risk margin. It will then be interesting to compare the value obtained for the Technical Provision with the market's value of the liability. The present value of the expected payoff from the option has already been calculated as €4.242. This is the best estimate of the liability in the Technical Provision calculation. Since the market in 1-year options is assumed not to be "*deep, liquid and transparent*" the option/guarantee creates an unhedged exposure. We will assume the capital to back this best estimate liability value is held in cash.

Let us assume that the 99.5% worst outcome over 12 months is a 35.9% reduction in the total return index in line with the true, known distribution for the total return index[6]. In practice, there is significant uncertainty around this estimate but it would be viewed as a plausible assumption by many practitioners. Of course, the assumption is more severe than the current, soon-to-be-reviewed 1-year equity stress under the Solvency II standard formula of 32%. We can now calculate[7] the 99.5% required capital at the end of the period as €31.471 which requires a start-of-period SCR of €29.936. Assuming this required capital runs off evenly over the year, applying the 6% charging rate we can calculate a risk margin of €0.898.

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[5] *It is an approximation because this is the option's <u>initial</u> beta. The option beta will vary stochastically in line with the theoretical replicating portfolio's exposure to equity.*

[6] *Since the real-world distribution is assumed to be log-normal this is exp(risk free rate + risk premium – vol^2/2 +N(1-0.995)*vol) – 1 = exp(.05 + .04 -.20^2/2 -2.576*.20)-1.*

[7] *The fall in the equity market over 12 months is offset by the accumulated value, at the risk free rate, of the best estimate value of the option. The net amount is then discounted to a start of period value at the risk free rate.*

This would make the total Technical Provision, when calculated using the Solvency II methodology described in CP 42, as €5.140. This value can be compared with the market's value which was €7.081. The market-consistent implied cost-of-capital for this particular market exposure is not 6% but 19%. In other words, if we apply the CoC approach using a 19% cost of capital rate, the 'correct' option price is obtained. The problem is, where market risk exposure exists, a model is required to discover a rational discount rate.

Strike	Market consistent value (MC)	Risk margin (RM)	Ratio (MC/RM)	Market-Implied CoC
0.60	0.059	0.006	9.8	Na
0.80	1.325	0.888	1.5	12%
1.00	7.081	5.140	1.4	19%
1.20	18.807	15.788	1.2	22%
1.40	34.739	31.580	1.1	21%

The values set out above compare the CP 42 value with the market's value for an at-the-money option. The calculations are repeated for a range of option strike prices (i.e. the level of the guarantee offered to policyholders) to obtain the table above.

A few conclusions are clear from the analysis:

- The risk margin values can be materially different from their market consistent equivalents. In this example, they all happen to be lower.
- The relationship between the two sets of values is not simple. As we saw in the first section of the note, the determinants of value for an option containing market risk can be quite complex.
- For the 60% strike, because the guarantee falls below the 99.5% stress, the CoC approach merely contains the PV of expected guarantee costs. The risk component falls away completely.

No doubt if we had chosen a different term and volatility (for example) combination, a different relationship between the risk margin values and their market consistent equivalent might have been obtained. The key point is that the risk margin does not appear to be a good way to value market risk.

The 6% cost of capital was not intended to cover the cost of bearing market risk i.e. the component we describe as MVM1. Even allowing for this, given that it is based on a view of frictional costs, it is doubtful that it will be consistent with the factors driving MVM2 – the jump and volatility risks and trading costs faced by holders of options on risky assets.

Finally, we should point out the convenient nature of the example we have used (where the horizon for capital calculation and product maturity are aligned). In practice the cost of capital approach will create challenges on a number of levels. Firstly, it may require a nested stochastic approach. Secondly, it can create circularity – the current capital requirement is being calculated based on assumptions about the behaviour of future capital requirements (which themselves will require assumptions about future capital requirements, etc). Turning the cost of capital calculation into a practical tool is a huge technical challenge for firms.

Conclusions

There are a number of important insights provided by the analysis. For a market option which carries exposure to general market risk, the present value of best-estimate cash flows (using a risk-free rate) can turn out to be quite different to the 'correct' option value which needs to be adjusted both for the correction in the discount rate given the systematic exposure of the cash flows and the hedging and capital costs of traders. For unhedgeable risks, the analysis suggests that we should aim to answer two sorts of question when thinking about MVMs. Firstly, is the cash flow exposed to any general market risk? If so, the discount rate should be adjusted (up or down) to reflect this. Secondly, what additional margin should be applied to capture capital and hedging costs?

Option prices – along with all financial asset prices – already contain MVMs. Option models have the capability to allow for MVMs in an economically sensible way which the cost-of-capital approach does not.

As a result, we strongly believe that for extrapolation of option prices, an option valuation model is the simplest means of producing economically coherent 'pseudo-prices' for missing maturities or strikes. Rather than attempting to discover the 'true' model for valuation, the pragmatic approach used by practitioners is to scale option volatility and other assumptions (although we would not use the *Black-Scholes* model for this purpose). Switching to a 'full' cost-of-capital approach in the presence of market risk will require the estimation of some elusive discount rates as well as creating some major technical implementation challenges.

Finally, it is important to stress that the use of models for marking non-hedgeable risks to market is by no means straightforward. For equity index options, firms still need to select suitable models and assumptions. Similar challenges exist in the pricing of long-dated unhedgeable interest rate options and the extrapolation of yield curves. However, in all these cases, a model allows insurance firms to impose economic consistency on valuations that is impossible to achieve using the cost-of-capital approach.

Disclaimer