In part two of this series, we looked at a curve fitting approach. We examined how it could be used to speed up the calculation of 1 year VaR capital by using ‘nested stochastic with interpolation’. In this article, we turn our attention to the least-squares Monte Carlo (LSMC) approach and some of its advantages over curve fitting – primarily increases in the speed and accuracy of calculations.

A basic LSMC capital calculation is similar to a full nested stochastic calculation. We will still do many thousands real-world (outer) simulations for 1 year. However, instead of running thousands of market-consistent (inner) scenarios to value our liabilities, in LSMC only one inner simulation is used.

Obviously using only one scenario gives a very inaccurate liability valuation at time 1 for each real-world scenario. We correct for this inaccuracy by doing a regression through these inaccurate valuations (hence the least squares name). The regression curve is then used instead of the inaccurate valuations to approximate the liability.

This regression is multi-dimensional: the values of the risk drivers at time 1 are the explanatory variables in the regression and the liability value is the response variable. The resulting regression curve gives us an approximation of the liability value as a function of the multiple input risk drivers.

There are different ways to do this regression but for simplicity we can think of fitting a polynomial function like the one we used in the curve fitting example given in part two of this series of articles. The goal is to find the coefficient values that give the best regression fit. Exhibit 1 illustrates the process.
Viewed from a different perspective:

This may seem an unlikely concept but it works! Exhibit 3 shows an LSMC projection for our example put option. For simplicity, we only consider equity risk and assume that all other variables are constant.
A few features of this chart are worth pointing out:

- The chart shows 2000 least squares Monte Carlo scenarios.
- The x axis is the value of the underlying equity at time 1.
- The y axis shows the put option value at time 1. (For a single scenario the put option value at time 1 is the discounted payoff in that scenario.)
- The green dots are very inaccurate put option values – using 1 market consistent simulation each.
- The spread of the dots over the x axis conforms to the distribution of real world equity returns over 1 year – there are lots around 1.1 (a 10% increase over the year) and very few less than 0.7 (a 30% fall). The starting equity value is 1.
- Although the individual dots show a broad spread of values, the general trend is for proportionally more of the inaccurate estimates to be higher where the starting level of equities was lower – this makes sense for a put option.
- The green line represents a polynomial regression through the dots.
- The true put option values are calculated analytically and shown by the blue squares. We can see that we get a very accurate approximation with the regression. This is especially impressive considering we are valuing the option at every point across the equity risk dimension with only 2000 simulations. A brute-force, nested Monte Carlo simulation would have used 2 million scenarios to get a similar result (with 1000 inner scenarios for each valuation).

The basic LSMC technique shown here has some room for improvement. Because of the small number of scenarios in the tail of the real world distribution, we can get large errors in the regression fit where accuracy is most important to us.
Here we see that while the central part of the regression curve retains accuracy under both examples, the fit in the left-hand tail is very sensitive to randomness in the scenarios. This can lead to over (or under) estimation of the SCR. (The SCR for this simple example will be the value of the regression function in the 99.5% worst equity value scenario).

To gain more accuracy in LSMC, we can split the calculation of capital into two stages – a fitting stage and a projection stage.

In the fitting stage we derive a liability function that gives the market-consistent value of liabilities over a broad range of values.

In the projection phase we pass a set of real-world scenarios through the liability function to produce a distribution of the liability value after 1 year.

This two-stage process has the added benefit that we can use more real world scenarios to calculate the capital than we have used to produce the fit. (So we can eliminate sampling error in the real-world scenarios without increasing the runtime of the ALM model.)

This fitting approach is illustrated in Exhibit 5.
The scenarios here cover a broader range than before; with many more in the left-hand tail range.

We have also used another trick to improve the fit: Each fitting valuation now uses two inner scenarios instead of one. Using two scenarios gives a more accurate valuation for each fitting point, reducing statistical noise and leading to more accurate regression fits. (Increasing the number of scenarios further gets us closer to a nested stochastic simulation). The number of fitting points is the same (2000) as in the previous examples but we now use 4000 scenarios as we have two simulations in each fitting point.

By using these two enhancements, the quality of the fit is dramatically improved.

We can draw parallels between this LSMC approach and the curve fitting approach. In both methods we fit a curve to some fitting points and pass our real-world scenarios through the curve to calculate capital. In the curve fitting method we fitted using a small number of relatively accurate points. In LSMC we fit the curve by using a large number of inaccurate points. The distinction is subtle but important.

In curve fitting we used the value of each fitting point independently of the others to fit the curve. Here we use every fitting point to inform the value of the full regression function. This gives a more efficient use of the information contained in our ALM scenarios and means we can find a more accurate function with less runtime.

As more dimensions of risk are added, the relative accuracy of LSMC over curve fitting increases dramatically.

In an SCR calculation with many risk drivers, where we need both speed and accuracy, the LSMC method is the clear winner.

There are other benefits to using the LSMC technique too.

**Modeling the First Year:**

In the first year of an ALM model, there can be complex dynamics that affect the value of liabilities at time 1. For example, even if the value of equities is the same in two scenarios, the value of liabilities may be different because the equity value took a different path to get there. This is a common feature of insurance models where bonus allocations and management actions are important.

LSMC can quite easily take account of these features by increasing the number of explanatory variables in the regression. For example, we can use more variables to describe the path of economic variables, model state variables like asset allocation percentages and bonus rates etc. Of course this is theoretically possible in curve fitting too, but each variable we add gives an extra dimension of risk. These extra dimensions multiply up the number of stresses required to fit an accurate function and make the technique unfeasible. In practical applications of the curve fitting
method, it is more common to assume some fixed path over the first year in each stress, making modeling of management rules and rebalancing strategies impossible or inaccurate.

It’s worth noting too that LSMC can be used equally well with a time 0 or time 1 liability stress definition. So we can make the assumption that stresses to liabilities occur instantaneously in the liability model with 1 year of economic risk. This can be useful when integrating the technique with other methods for a group wide capital calculation.

**Calculation of standard errors:**

Using LSMC allows us to estimate the standard error of our SCR – we can estimate how much potential random error our calculation may have and how many scenarios are required to bring the accuracy to the required level.

In the case of curve fitting, calculating a standard error is more difficult. (Although it is possible to calculate the standard errors of our fitting points, we can never be sure how much error is introduced by interpolating the function between fitting points in many dimensions.)

**Non-market Risks:**

We can include non-market risks in our LSMC calculation. It’s a simple case of adding scenarios for non-market risk drivers to the fitting scenarios and running these through the ALM as normal. The regression function will be created to be a function of market and non-market risks. These could be persistency rates, mortality rates or other risks.

The real-world scenarios will also have to include scenarios for non-market risks. (The distribution of non-market risks must be specified – or will be implied – by any SCR calculation method chosen.)

This scenario-based method for non-market risks can also be used in a curve fitting calculation but again, the increase in the number of stressed points makes this impractical and in practice much approximation is used and error introduced.

**Robustness:**

The LSMC technique is robust. It can be used across a number of different business lines and over many economic regimes and changes in liability features. For example, if we added some complex hedging assets or management actions to our liability books, this may create non-monotonic or other odd looking liability functions. These are automatically handled by LSMC because the flexible regression line fits changes to fit the full range of scenarios. If we were using curve fitting we would need to re-evaluate the number and location of our fitting points to be sure we still captured the shape of the liability function and were able to interpolate accurately. This is a cumbersome process requiring many more runs and creates the risk of delays and inaccuracy in the reporting process.

**Rolling forward the capital calculation:**

After the calculation reporting date, we may have the need to re-calculate the capital requirement. For example if equity markets fall 10% in the week after the calculation date, we may quickly want to know the effect on our liability and capital values. Instead of re-doing the full calculation it would be nice to roll forward the last calculation.

This is easily done with LSMC.

Because we choose our fitting scenarios to span a very broad range of each risk, and because of the accuracy of the function over the full range, we can simply update our initial position and real world scenarios and re-use the same function. This is a very quick exercise because it does not require new ALM runs.

With a curve fitting approach this would require many more fitting points to cover a broader range (the range we should aim to cover with our function is a 99.5% stress
after a 99.5% movement in all variables). It would also require more intermediate stresses in each risk dimension to give a better level of accuracy in the interpolation as this will affect our starting position and the subsequent SCR calculation.

**Calculating future capital requirements:**

As well as doing an accurate calculation of capital today, we want to know how our capital behaves in the future.

This may be to complete an Own Risk and Solvency Assessment (ORSA) or to do return on capital or risk margin type analyses.

Here we see a major benefit of LSMC.

Recall that in the LSMC technique, we did a regression to get a liability function at time 1. We can also do this regression at any future year. This gives the liability function at every future year.

**Exhibit 6** Least Squares Monte Carlo fit to a put option through time

This shows how we can do a regression through the same set of fitting points in different future time periods (shown in different colours).

Again the regression fits compare very well to the true put option values. (Here the liability function at time 2 will be a 9 year put option, time 3 an 8 year option etc.)

Where actuarial models are more complex than the simple put option considered, some work may be required to get the future liability functions fitted correctly – particularly where the liability values in future years depend not only on the latest value of economic variables but also the path taken to get there. These path-dependent features can make the liability function complex to fit because it may depend on the full history of economic variables. A successful regression through time will depend on identifying state variables in the model that summarize the features of the historic path that are relevant to valuing the liabilities.

Again this type of fit is theoretically possible using the curve fitting method but the number of stresses required multiplies up by the number of future time periods fitted. The problems of specifying the path become far more complex and reduction in accuracy larger. This would not be a practical method beyond year 2 and even then would hang together poorly.
Wider application:
The multi-year fit provided by LSMC gives the method broader application in the field of insurance work than a capital calculation. These include hedging, pricing, definition of management actions, asset allocation optimization. The prospects are enough to make even the most hardened actuarial modelers giddy with excitement.

Summary
In summary the speed, accuracy, flexibility and robustness of LSMC results in models that can actually help – rather than hinder – intelligent decision making in insurance companies.

In the past companies have often had to rely on incomplete and inaccurate analyses to support decisions due to unsuitable models. Good decisions in choosing a Solvency II modeling framework will mean better functioning companies. Bad decisions will invite old problems for many years to come.

In this article we have seen how LSMC can be seen as a more efficient and useful implementation of curve fitting and can dramatically reduce the runtime of nested stochastic models to the point where they are feasible for use in decision making.

In the next article in the series, we will consider the Replicating Portfolio technique for SCR calculation.

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