Replicating portfolios to calculate capital

In a previous article, *A primer in replicating portfolios* in this series, I explored a set of useful principles for replication — general facts that govern the use of replicating portfolios in asset-liability modelling. I continue the series by exploring the practical aspects of RPs in insurance capital calculation.

**Capital calculation requirements**

Before looking at current practice, it is useful to set some objectives by which we can measure the efficacy of the RP method:

- Finding the RP on a regular basis should be quick, methodical and automated.
- The RP should be quick to revalue.
- The RP should be accurate in the scenarios that drive capital.
- Preferably, the RP should be accurate in all scenarios so that we can derive a full distribution of MCEV in one year.

We should be able to verify that it is accurate. As we step through the RP process, looking at some common methods in use, we will keep these criteria in mind.

**Current insurance replicating portfolio process**

The overwhelming majority of replicating portfolios in the insurance world to date have used static replication. The assets in the replicating portfolio are assumed to be held to maturity and the weights are chosen to match the cash flows of the liabilities as closely as possible. No rebalancing takes place. For the remainder of this article we’ll use replication to mean this, fully static, variety.

The general process for creating a replicating portfolio in practice is:

- Choose a set of candidate assets for the RP
- Run a set of economic simulations through the liability model
- Run a fitting algorithm to find the weights of the candidate assets in the RP that give the closest match to the liability cash flows
- Test the fit of the RP by valuing the liabilities and the RP under stressed conditions
Looking at each of these steps in turn:

**Choosing assets**

Ideally we would only include assets that can be easily priced in our RP; whose prices can be readily observed in the market and whose behaviours are well known and easily communicated. This will reduce our complex liabilities to a simple market portfolio that can be easily administered on an ongoing basis. We would like to avoid assets that need Monte Carlo simulation to value.

We can then follow the principles we have already laid down:

To start with, we need to use assets that are exposed to the same underlying risk drivers as our liabilities. So, if we have guarantees on the value of equities in our liabilities, then we should include equities and equity options in the candidate assets. If we have fixed cash flows, then we should use bonds, and so on.

This may sound simple enough but recall that:

- To replicate complex liabilities with multiple risk drivers and path dependency, we need to include assets that have these features.
- To be more efficient with the number of assets we use, we should include assets with pay-offs that are similar to our liabilities.

Many of the liability books which we need to model in practice embed combinations of some or all of the following features simultaneously:

- Dependence on multiple underlying assets
- Options based on equities, interest rates, credit, currencies
- Book value accounting
- Guarantees on regular premiums
- Options on options
- Regular profit lock-ins
- Options for the policyholder to exercise their guarantee early
- Options for management to change the guarantee level
- Options for management or policyholders to change the assets on which the guarantee is provided
- And many others

These are defining features of insurance liabilities and are embedded in the ALM models that generate their cash flows.

Trying to replicate these features using only simple assets will often not converge to the correct answer and may lead to the following problems:

- **Globally poor replication**
- Replications which produce correct sensitivities to some risk drivers and absurd divergence for others
Unstable replication quality from one reporting period to the next

Replications that take weeks of manual manipulation to produce even ‘acceptably bad’ fits.

The degree of error introduced varies by business type and over time.

Including well known multi-asset or path-dependent assets that are not frequently traded can improve the replication. However, we may still encounter problems. It can be difficult to find assets that include a wide enough range of economic risk drivers simultaneously and exhibit the right path dependence characteristics. Even if we can find them, we may require many thousands of options to form a complete basis for replicating a complex set of liabilities.

In general, it is extremely challenging to apply RPs well for even liability books with moderate complexity using only simple asset types.

Creating assets for replication

Luckily we are not limited to using recognised assets in our replication. We could create our own assets that have features similar to the liabilities. This should give a more accurate and elegant solution to the problem with fewer replicating instruments.

These exotic assets should depend on the same asset returns, have the same guarantee features and as far as possible the same internal rules. The result is improved accuracy with far fewer assets, making the process far easier and more accurate for books with complex liabilities.

But why bother with exotic assets?

If assets can be created that work better for RP fitting, policy model points from the underlying liability book should be the best instruments of all. We can choose a very small subset of policies that represent our liabilities well. These will automatically have the latest liability rules and features embedded and form an intuitive portfolio.

The problem with including even simple exotic asset or model points is that they cannot easily be priced using closed-form solutions. Their pricing almost always requires a Monte Carlo model – so it doesn’t remove the nested stochastic problem. However, using a reasonably small set of instruments should result in manageable run times with reasonably inexpensive hardware resources.

In practice, faced with this conflict, practitioners have usually opted for simpler assets, and compromised on accuracy.

Other advisers have suggested an incremental approach to RP implementation – start with a simple set of assets and evolve into more complex ones if necessary. This may be a Trojan horse – and a very expensive way to create a set of model points.

Fitting scenarios

Once we have chosen a set of candidate instruments, we run a set of economic scenarios through the asset-liability model and the RP.

We saw in the previous article how a grid covering a large range of possible economic conditions would be useful to ensure we have enough information about the liabilities to do the fitting. We need to know about the cash flows under extreme changes in the risk drivers. We also need to know how they behave under path dependency: for a given asset loss, will the liability cash flow differ if the loss happens in the first year rather than the last year?
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In practice, insurers will usually use a set of Monte Carlo fitting scenarios instead of a grid or tree-based technique. We can think of the creation of fitting scenarios as shedding light on a multi-dimensional liability shape – we want to find out the form of the liabilities in all dimensions and over all relevant regions.

The problem here is that large regions of the space are unexplored. There are only a small number of simulations where both assets experience losses (which are of most interest to us for capital calculation). There is also no exploration of the result of instantaneous changes in the value of assets.

One method that has been used to try to rectify these pitfalls is to use more than one scenario set, for example, a base scenario set plus a stressed one.

This gives us more scenario outcomes in the region that we think drives capital. However, there is still not much illumination of other regions of the space. We don’t have any information about the effect of other asset stresses at time 0 and little or no information about the effect of large increases in either asset on cash flows. In more than two dimensions it would also be difficult to choose the initial points and cover a large amount of space.
This method allows us to cover all risk drivers over a pre-specified multi-dimensional fitting space.

**Finding weights**

Once we have the sets of RP asset and liability cash flows, we need to find the weights of the RP instruments that give the best fit to liabilities. The easiest method is an ordinary least squares regression: we find the asset coefficients that minimize the sum of squared difference between the cash flows of the RP and liabilities.

As before there are complications:

- We often include thousands of assets in the RP, making the regression difficult.
- We often include many similar candidate assets in the RP. This causes mathematical problems related to co-linearity in the regression.
- Where we have assets that are not able to fit to the liabilities, good cash flow fits do not imply agreement between the value of the RP and the value of the liabilities. Models that do not converge well like this are difficult to control, predict and automate. This is almost always the case when simple, traded assets are used.
- A small number of extreme scenarios (with large squared errors) can have a large influence on the fits, leading to bad fits in important regions. This is exacerbated by the point above.
- Where we have a small number of scenarios relative to the number of candidate assets, it is often possible to over-fit. This gives RPs which fit the cash flows well but with very poor predictive power.

Solutions to most of these problems can be found with more sophisticated fitting algorithms, such as that shown in exhibit 4 below.
In this case, we use half the scenarios to choose the weights of the assets in the RP and the other half to test if the assets perform well out of sample. By doing so, we can avoid including assets that only serve to explain random noise in our scenarios.

Less scientific rules of thumb for performing the fitting are also commonly used. For example in creating and weighting fitting scenarios, choosing the optimisation technique and the objective measure.

The end objective is to maximize some statistical measure of the agreement of the cash flows such as the out-of-sample $R^2$. This measure is useful for the fitting but is not a good guide to how accurate the RP is likely to be for a capital calculation. For this we need to perform some validation of the RP.

**Measuring accuracy**

We are interested in how closely the stressed value of the RP matches the stressed value of the liabilities. To test this we may perform a valuation of both, using the same set of Monte Carlo scenarios, and compare the results. These values should agree closely.

When sets of market-consistent scenarios are used for the fitting, it is common to use these same scenario sets for validation. However, this leaves a different kind of in-sample problem. It arises because the initial stresses we have used for the fitting and testing are the same.

To be truly thorough in testing, we should perform complete out-of-sample tests, choosing initial values for testing that are different from the fitting ones.
These valuation differences should not exceed some pre-defined accuracy tolerances. If we do not manage to achieve these targets, we are often stuck. With the tightness of reporting cycles there is usually not sufficient time to define new exotic assets.

The unfortunate junior actuary is left to tinker into the small hours of the morning. Trial and error choices of fitting scenarios, scenario weights, optimisation techniques and objective functions are often employed to improve the agreement of the RP to the testing valuations. This is a seldom acknowledged form of in-sample fitting. The RP is manually altered through optimisation levers until a small number of pre-selected tests are satisfied. The result is a fit that is compromised in other regions that are not well tested.

This is commonly called the art of replication, but these manual processes often mask underlying problems with the replication process to do with instrument selection and choice of fitting scenarios. They also make the process difficult to automate to the degree required in modern financial reporting.

**More fundamental problems**

Aside from challenges in the fitting of economic risks, RPs face more fundamental problems in insurance work.

A distinguishing feature of most liability books is the simultaneous dependence on market and non-market risks. A simple example is annuity policies. These contracts contain interest-rate and longevity risks. If interest rates decrease, we may lose money through holding assets of shorter duration than our liabilities. If people live longer, we lose money because we need to pay out for longer than we anticipated. But when both changes occur at the same time, we lose more than the sum of the parts – because increasing life expectancy increases the duration of our liabilities and hence the sensitivity to interest rate changes.

We could replicate the interest-rate sensitivity quite easily with an RP made of fixed-interest assets. But it is not possible to capture longevity sensitivity or the joint dependence when using market-traded or even exotic financial instruments.

These types of risks are ingrained in almost all practical insurance problems in the form of mortality/longevity assumptions, lapse rates, management actions and policyholder decisions. They are explicitly recognised by the Solvency II standard formula as necessary risk factors to stress and can have large joint effects with financial risks.

Including these non-market risks requires us to give up on any attractive features that could be gained from RPs with market traded assets and accept a nested stochastic model with a reduced set of liability model points for calculating capital.

**Wider uses**

RPs have also been proposed for use in wider fields than capital calculation. Unfortunately these uses suffer from similar problems to those described above and also some new ones.

When employing RPs for hedging, differences in cash flow fits are often magnified in differences between portfolio and liability values. This can give bad results for analysing long-term static hedges. These inaccuracies are then exaggerated again in the calculation of Greeks for dynamic hedging – leading to unstable and costly programmes.

RPs have also been proposed for use in asset-allocation decisions. However, they do not account for the interaction between assets and liabilities within the insurance portfolio. For example, when we change the assets in a participating insurance book, the liability value
and risks change too – there is a feedback loop that does not exist in the RP. (Changing the assets in the RP in a similar way – e.g., replacing equities with bonds and equity options with swaptions will not have the same effect as similar changes in the liability model.)

Differences in department, specialisation, location and priority between the creators and users of RPs are especially dangerous for these applications. The fact that the RP is employed in regulatory capital calculations may give spurious confidence in its accuracy and broader applicability.

**The many practical problems**

We have seen in theory that replicating portfolios can match a set of liabilities. We have also seen how useful they can be if they are able to match the liabilities accurately.

Unfortunately, we have also seen how their practical use may carry many problems:

- The inability to converge accurately to financial liability values.
- The inability to account for non-market risk.
- The requirement for large numbers of fitting and validation scenarios.
- A propensity to over-fitting and user manipulation.
- A process that is unstable over time and is difficult to reliably automate.
- The necessity of using exotic/model point type instruments that cannot easily be priced without the use of Monte Carlo models.

For these reasons, replicating portfolios continue to be seen by many people as unsuitable for accurate, timely, consistent and automated capital calculation and reporting.
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