



Steffen Sorensen

## One year calibration and choice of equity model

For a one year projection of equity returns we have a choice between a simple log normal constant volatility equity model and more complex models such as the Stochastic Volatility Jump Diffusion model where the return distribution is skewed and has fat tails. While we want the simplest possible model to capture features of equity returns relevant for a forward looking projection we want to avoid underestimating risks. Historical data and, more recently, implied volatility from option markets, suggest that:

1. Equity volatility changes through time but eventually reverts to its long-term level.
2. The distribution of returns is skewed and has fat tails.

The evidence is stronger the higher the frequency of the historical data. If these are permanent features of equity returns, it can have important consequences for choosing a simple model for a (say) one year projection as required for Solvency II. We illustrate this comparing the projection from the two equity models against realised returns in 2009. Had we used a constant volatility log normal model, calibrated to a 'through-the-cycle' target for return volatility, we would have underestimated the risks.

### Equity return volatility

Over the past 15-20 years a liquid exchange-traded option market has emerged from which we can back out an estimate of market expectations of equity return volatility. Exhibit 1 compares one month return volatility from at the money option markets against realised return volatility over the following month for Euro Stoxx 50. The difference between the two is one measure of unexpected volatility.

The conclusions from the two series are common across international equity markets. First, return volatility changes through time but will revert to its long-term level over time. Second, option-implied volatility is, on average, higher than realised volatility. This can be attributed to a volatility risk premium required by option holders to compensate for unexpected volatility. This is because equity volatility is not fully anticipated; it changes stochastically through time.

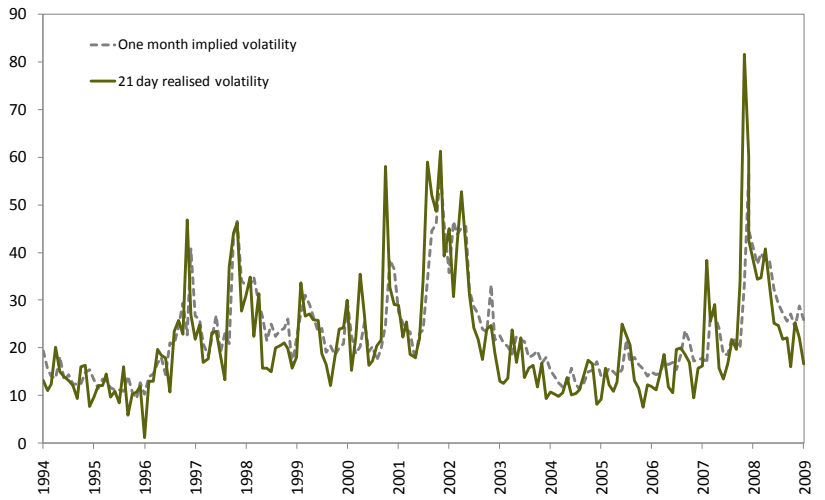
Any real-world volatility term-structure target therefore changes from one calibration to another. Exhibit 2 shows our target volatility term-structure for the Euro Stoxx at end December 2008 and 2009. The target over a one and two year real-world equity return projection can change materially from one calibration to another. The mean reversion observed in return volatility, however, implies that the volatility projection at medium to long horizons changes less. **In a constant**

volatility log normal equity model, the volatility term-structure is flat and not stochastic. This makes it less suitable for a real-world equity return projection if not calibrated carefully.

*.....market implied volatility tends to be higher than realised equity volatility because option holders require compensation for unexpected volatility*

**Exhibit 1**

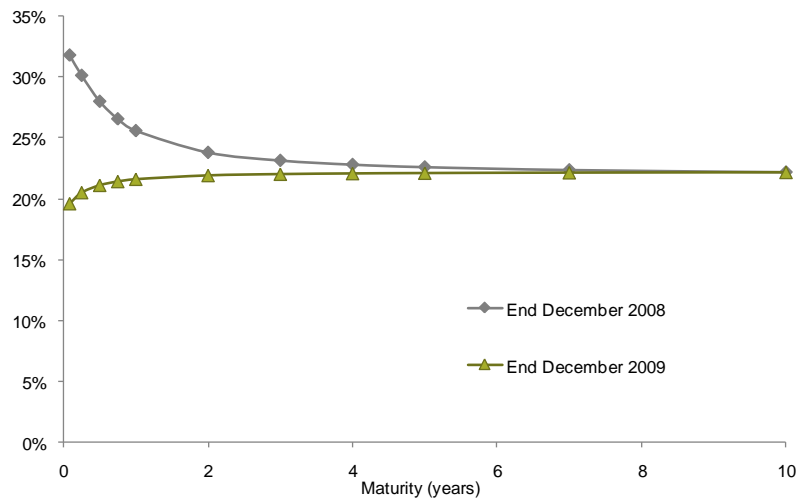
Euro Stoxx 50 one month 'At the Money' equity implied volatility and 21 day realised equity volatility



*.....short term real-world equity return volatility does change over time but will eventually revert to its long term level*

**Exhibit 2**

EUR Real-World target volatility term-structure at end December 2008 and 2009

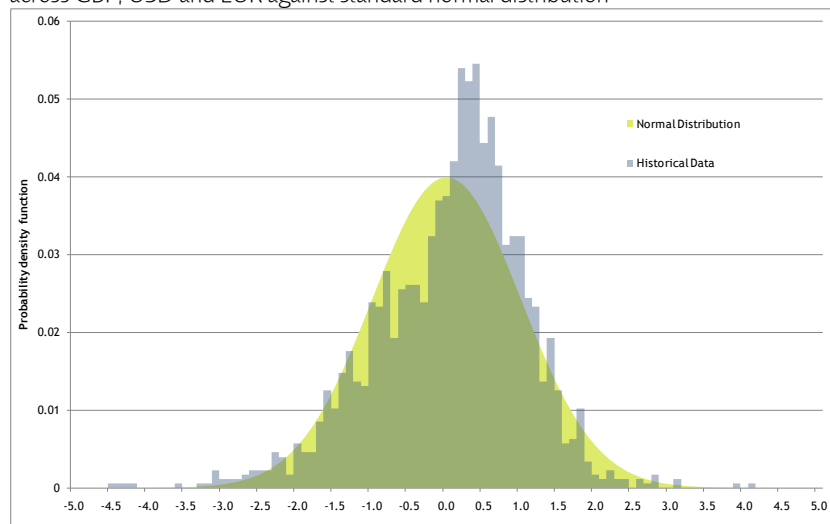


Stochastically changing equity volatility suggests that the distribution of returns may deviate from a simple log normal distribution. Indeed our research, and consensus among other researchers, suggest that the shorter the projection horizon of equity returns, the larger the departure from log normality. The distribution of equity returns at a projection horizon of one month and one year departs significantly from the log normal distribution. Historical data at such short time intervals tend to exhibit skew and excess kurtosis. In other words the downside risks are higher.

**Exhibit 3** shows the standard normal distribution against the empirical distribution from a pooled data set of annual log return across the Euro area, UK and US since 1960. **Note the significant departure from the normal distribution in historical data.** To obtain a return distribution that shares some of the features of the historical distribution we need more advanced models such as the SVJD model which can be calibrated to the skew and kurtosis observed in historical data. With a constant volatility model there are limited degrees of freedom and care should be taken if calibrating that model.

### Exhibit 3

Standardised empirical percentiles from pooled one year equity annual return data across GBP, USD and EUR against standard normal distribution



**.....historical data suggests a significant departure from log normality of equity returns**

## The choice of model calibration is important

Does the choice of model matter and to what extent do we underestimate the risks when using a constant volatility log normal model for a one year real-world return projection?

**Exhibits 4 and 5** show the projection of equity return at end December 2008 for 2009 against realised outcomes for the Euro Stoxx.<sup>1</sup> The constant volatility model is calibrated to a long-term ('through-the-cycle') view on equity volatility whereas the SVJD is calibrated to a term structure of equity volatility target and our targets for skew and kurtosis in the equity return distribution. We could obviously have calibrated the constant volatility model to a specified percentile target or a one year short term ('Point-in-time') volatility target. But the volatility term structure in the model would be flat and the distribution of return symmetric.

Two observations stand out:

- 1) We notice that the 'through-the-cycle' constant volatility model assigned very low probability to some of the outcomes realised in 2009.
- 2) The return distribution is symmetric. In the Stochastic Volatility Jump Diffusion model the return projection has a volatility term structure and exhibits skew and fat tails. A higher probability is assigned to extreme negative returns relative to extreme positive returns. Because the log normal equity model fails to capture the volatility term-structure of equity returns and the skew and kurtosis observed in historical equity returns, it is a less suitable model for real-world projections of one year or less. Given the limited degree of freedom in the constant volatility model there is a difficult calibration choice

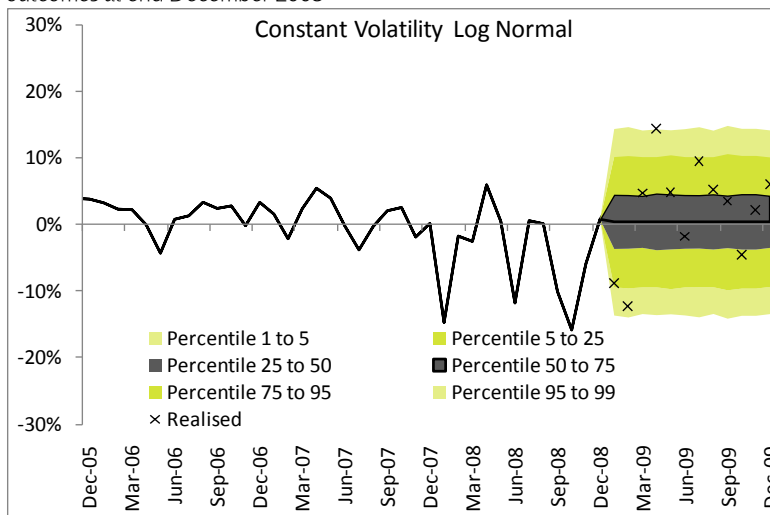
<sup>1</sup> See Barrie & Hibbert financial market update, end December 2008.

and a material danger that we may end up underestimating risks. Given its flexibility, the SVJD model can be calibrated to more realistic features of the equity return distribution and it is therefore a better model for a one year calibration. Other advantages of the SVJD model include the ability to calibrate the model to capture stronger tail dependence, as observed during 2008, across international equity markets.

.....the distribution of equity returns in a constant volatility log normal model is symmetric

**Exhibit 4**

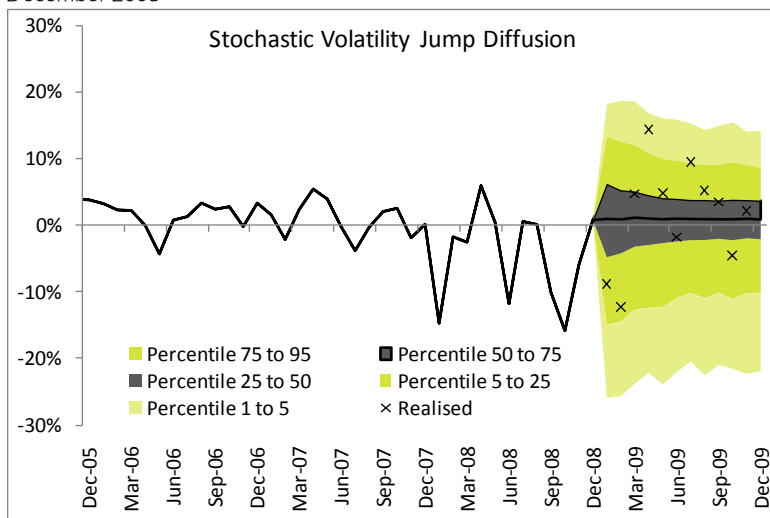
One year projection from constant volatility log normal equity model against realised outcomes at end December 2008



.....the distribution of equity returns in a SVJD can be calibrated to a volatility term-structure, skew and kurtosis as observed in historical data

**Exhibit 5**

One year projection from SVJD equity model against realised outcomes at end December 2008



## Disclaimer

Copyright 2010 Barrie & Hibbert Limited. All rights reserved. Reproduction in whole or in part is prohibited except by prior written permission of Barrie & Hibbert Limited (SC157210) registered in Scotland at 7 Exchange Crescent, Conference Square, Edinburgh EH3 8RD.

The information in this document is believed to be correct but cannot be guaranteed. All opinions and estimates included in this document constitute our judgment as of the date indicated and are subject to change without notice. Any opinions expressed do not constitute any form of advice (including legal, tax and/or investment advice).

This document is intended for information purposes only and is not intended as an offer or recommendation to buy or sell securities. The Barrie & Hibbert group excludes all liability howsoever arising (other than liability which may not be limited or excluded at law) to any party for any loss resulting from any action taken as a result of the information provided in this document. The Barrie & Hibbert group, its clients and officers may have a position or engage in transactions in any of the securities mentioned.

Barrie & Hibbert Inc. and Barrie & Hibbert Asia Limited (company number 1240846) are both wholly owned subsidiaries of Barrie & Hibbert Limited.