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Market-consistent valuation of ultra long-term cash flows

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Surely, there is no more fundamental task in financial management than placing a value on a certain future cash flow? Yet this basic task turns out to present a major challenge to economists and financial analysts and continues to be the subject of a heated debate among accountants, regulators, actuaries and firms' CFOs.

In this note we will explain what can make this simple question so difficult to answer as well as highlighting some of our own work which is aimed at helping to build a consensus on what a reasonable set of answers could look like. Note that, while our focus is on financial entities carrying known liabilities, these ideas apply equally to the valuation of any certain cash flow.

Today's prevailing view (with which we agree) is that information from financial markets can and should be used to value risk-free cash flows [1].So, if the market price of a future risk-free cash flow is $1, then this is the value that should be reported on the balance sheet. This is a simple, intuitive idea (at least for economists). However, it turns out to be far more difficult to apply in practice than you might imagine. Trouble comes from two directions: First, there are a lot of different instruments that promise known cash flows in the future (bonds) but they trade at *different* prices. It is not clear which is the most appropriate price for valuation. In the past we have discussed the merits of government bond or swap curves as a measure of risk-free interest rates. That is *not* our focus here. Second, for many currencies, and for *all* currencies at certain future dates, market instruments offering a certain or near-certain cash flow simply don't exist[2] . In highly-developed fixed income markets we may be able to observe government bonds or interest rate swap contracts with maturities of up to 50 years. In less developed markets liquid bond quotations might be limited to only a few years. In exceptional circumstances there may be no traded risk-free instruments at all. Of course, the liabilities of insurance firms and pension funds

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[1] *Of course, many of the cash-flows firms seek to value are not certain or even near-certain. Valuing these cash flows (and estimating required risk capital in financial intermediaries) creates another long list of interesting challenges which is not our focus here.*

[2] *The problem of 'missing' market prices is not confined to risk-free interest rates. The creation of 'pseudo'-prices for fair valuation work turns out to be a major challenge for insurance and pensions valuation work since active markets are not observable in either the complex state-contingent claims held or simpler options with the required term and underlying asset exposures.*

frequently extend beyond the term of available market instruments. You can see that this creates two fundamental challenges. First, how to choose a set of sufficiently liquid market instruments (government bonds and/or swap contracts), collate prices and then infer the prices for all maturities spanned by the market. Analysts typically fit a yield curve over the data. It may be desirable to impose some restrictions on the smoothness of this curve.

Second, in order to value ultra long-term financial claims, practitioners must ***extrapolate*** yield curves to generate a set of 'pseudo-prices' for the inferred prices of discount bonds beyond the term of the longest available traded cash flow. In this note, we will focus on the question of how to extend yield curves in a sensible way. We will work with the most basic elements of the yield curve – ***forward*** interest rates[3] .

3 Questions

Extrapolation requires us to face three questions:

1. What is the longest market forward interest rate we can observe?
2. For the purposes of extrapolation, what is the ultimate very long-term 'unconditional' forward interest rate?
3. What path should be set between the longest market rate and the unconditional forward rate?

There is a further consideration which you should keep in mind. If you are going to perform this calculation more than once (we certainly will), you will be interested in ***changes*** in the extrapolated curve and discount bond prices over time. It is important to answer the questions in a way which looks credible at a single point in time ***and through time***.

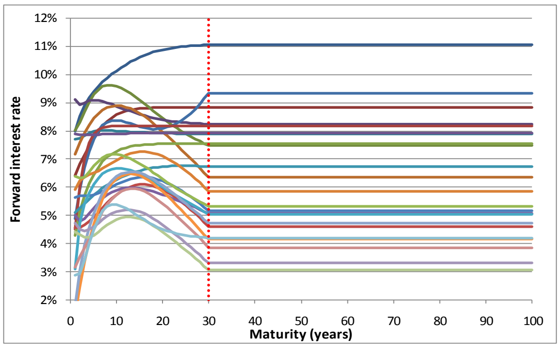
Policymaker's answers

These are not simple questions. Regulators and policymakers have addressed them in different ways. The CFO Forum establishes a general principle by saying that yield curves "*should be extended using an appropriate methodology*"[4]. You cannot really argue with that. However, they then offer some example methods which we believe will be appropriate in some circumstances but could fail elsewhere. In the most recent technical guidance for Solvency II, yield curve *'interpolations and extrapolations are based on the assumption that forward [interest rate]s are constant'*[5] . Bitter experience of dealing with worldwide yield curves suggests that, whilst this simple rule will work fine at some times and in some places, its universal application will produce results that we cannot justify.

Consider an example. The chart opposite shows USD government forward rates (estimated by Fed researchers[6]) at year ends between 1985 and 2007. Let us extend the curves using a constant forward rate at the longest maturity

Exhibit 1

USD government forward interest rates assuming constant rate beyond the longest maturity (30 years), end-December 1985-2007



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[3] *Although we are really interested in discount bond prices (the price today of a certain dollar to be paid at some specified time in the future), since it turns out that the price of a 30-year and 29-year discount bond are connected by the 1-year forward rate 29 years from today, it turns out to be easier to work with forward rates.*

[4] *CFO Forum: "Market Consistent Embedded Value Principles", June 2008.*

[5] *QIS4 Technical Specifications (MARKT/2505/08), March 2008, page 245.*

[6] *See Gurkaynak, Sack and Wright, 'The U.S. Treasury Yield Curve: 1961 to the Present', Federal Reserve Board Discussion paper 2006-28, http://www.federalreserve.gov/pubs/feds/2006/*

– 30 years in this case. You can see that some of the extended curves look quite reasonable but others don't really fit well with the market data.

Exhibit 2

Discount bond volatility for forward rates extrapolated from various terms at a constant level

Longest maturity for extrapolation (years)	*Term of discount bond (years)* 30	50	70	100
30	21%	38%	58%	90%
20	22%	33%	45%	63%
10	27%	44%	61%	87%
5	33%	55%	77%	111%
1	48%	80%	112%	161%

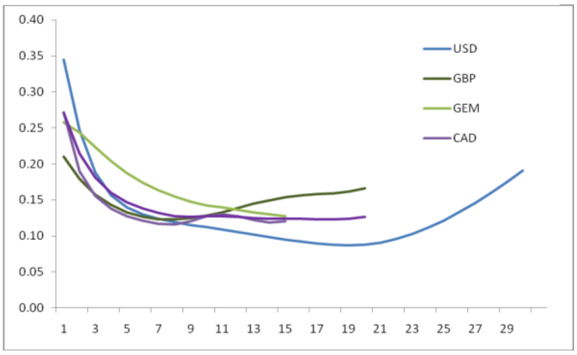
You can also see that any variation in the longest forward rate (at 30 years in this case) is transferred to all extrapolated forward rates. This will make the 100-year discount bond ***price*** pretty volatile. It turns out to have a volatility of 90% pa – approximately 5 times the long-term volatility of the US equity market. This might be a sensible assumption, but we would expect to take some considerable care before adopting it. Further, if we choose to transfer interest rate variability from a short–term maturity to longer maturities– and this is exactly what we will need to do when this method is applied in most emerging economies – the volatility of extrapolated discount bond prices could be far higher. The bottom rows of exhibit 2 show how the extrapolated bond prices will behave if we were to extrapolate the USD government forward curve at a constant rate from shorter maturity points. For example, if we extrapolate from 5 years at the 5 year forward rate, the 50-year discount bond has a ***price*** volatility of 55%.

Let us add a word of warning. Long-maturity forward interest rates are difficult to estimate accurately and it is worth highlighting a couple of issues. First, forward rates depend on the relationship between the prices of bonds at adjacent maturities. It turns out that small observation 'errors' in the prices of long-term bonds translate into big changes in long-term forward rates[7]. Second, the statistical methods used by central banks to estimate forward rates do not usually do very much to impose stability on the long end of the curve – they aim to get the best fit over a spectrum of traded maturities (typically to 10-15 years) at a particular point in time. As a consequence, these fitted long-term forward rates can be quite unreliable and volatile. The chart opposite plots volatility for forward rates estimated by the statisticians at four central banks. Many economists expect these curves to slope downwards (because of mean reversion in interest rates). You can see that for the US and the UK, apparent forward rate volatility falls for a time then ***rises*** at long-term maturities.

Exhibit 3

Historic volatility of government log forward rates

(1987 to date)



So, estimates for long-term forward rates may not be reliable and could exhibit spurious volatility. If we really believe this, it would be a pretty poor rate to choose to extrapolate into the far future. This would be true for both a point-in-time valuation and for updating the extrapolation. Of course, we should also point out that this behaviour could be entirely real – no mere statistical effect. Let us now take a look at the factors that influence nominal forward rates and suggest a different approach to extrapolation.

What determines very long-term forward rates?

We described forward rates as the basic building blocks for constructing yield curves. Economists break the nominal forward rate into a number of elements:

- An expectation for the real short-term interest rate.
- An expectation for inflation.
- A risk (or 'term') premium. Most practitioners believe that risk premia don't simply vary with term, rather there will be certain maturities that offer higher (or lower) expected returns.
- A technical 'convexity adjustment.

Exhibit 4

Assumptions for the elements of unconditional nominal forward interest rates

Assumptions for 2008	Government bond calibration	Swap calibration
Expected real interest rate	1.8%	2.0%
Expected inflation	2.4%	2.4%
Expected short rate	4.2%	4.4%
Term premium	1.5%	1.7%
Convexity adjustment	-0.4%	-0.4%
Unconditional forward rate	5.3%	5.7%

For the unconditional forward rate (UFR), the expectations will be ultra long-term expectations. The table opposite gives our economists' assumptions for these long-term elements of the UFR. You can see that the aggregate assumed

[7] *By 'error' here we mean effects like timing differences in traders updating prices of different issues or even individual bonds trading closer to bid or offer prices at a result of a trader's inventory position. These effects may be especially marked at (low-liquidity) year end dates.*

unconditional rate is 5.3%. We expect this to be a very slow-changing assumption because it is based on very long-term behaviour of investors, financial assets and inflation. It provides an alternative answer to question 2 posed above. It is certainly consistent and stable and its use as a long-term anchor could help to avoid the transmission of spurious volatility to long-term forward rates.

However, we have said nothing about ***when*** this rate might be reached. Is it reasonable to assume that such a limiting UFR is reached after 50 years, 100 years or 1000 years?

How should we extrapolate to a slow-moving unconditional forward rate?

The third question we posed above was ***'What path should be set between the longest market rate and the unconditional forward rate?'*** Should this be a slow or a fast adjustment? Evidence suggests that expectations for real short rates and inflation change only very slowly for horizons beyond 10 years[8]. It seems that – like most financial assets – variability in required risk premia accounts for a major part of asset price volatility[9]. The really interesting question is: how far into the future do these risk premium variations extend? If the risk premium embedded into a 20-year forward rate changes, how far is it reasonable to adjust the 50-year, 100-year and 1000-year forward rate? We do not believe they move by the same amount, so it's not appropriate to adjust extrapolated rates naively in line with those flaky, hard-to-measure longest market forwards. If you agree, you would also expect forward rates to be less volatile at longer maturities.

Let us make another point. For the purposes of valuing very long-term cash flows, reducing forward interest rate volatility seems like a pretty pragmatic approach. Do you really think it is appropriate to transmit all of the volatility observed in 10-year forward rates into the 100-year forward rate and thereby into an untradeable 100-year cash flow? We don't.

This question provides an interesting insight into the debate around fair value and mark-to-market. Market risk premia do appear to vary dramatically over short periods. Investors and regulators have now woken up to the fact that the variability of valuations depends directly on variation in asset risk premia (old-fashioned fear and greed).

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[8] *See for example Kim and Orphanides, 2005: Term Structure Estimation with Survey Data on Interest Rate Forecasts, Finance and Economics Discussion Paper 2005-48, Board of Governors of the Federal Reserve System.*

[9] *The realisation that changing risk premia account for so much asset price volatility is the source of so much discomfort among practitioners with 'fair values'. Whilst this has been recognised as a feature of equity valuations for many years the recent mark-down in credit-risky assets is now increasingly recognised as a consequence of a reappraisal of the required risk and liquidity premia on credit assets (in addition to changing expected – i.e. default-adjusted – cash flows)..*

So, can we do any better? How do we propose to set the decay profile for forward rate volatility?

So, is there a better alternative? We think so. Let us set a profile for the forward interest rate variation we expect to observe over time and then extrapolate in a way which is consistent with this. If we adjust the forward rate quickly to a slow-moving UFR, interest rate volatilities will decay quickly. If the adjustment is slower, forward rate volatility will persist for longer. We set targets for volatility decay which we believe are consistent with:

- Observed yield curve behaviour
- Economic beliefs
- Market prices for interest rate volatility

Our analysis is set out in the forthcoming research note on forward rate volatility decay[10]. The targets are plotted in exhibit 5 with red discs. We have also plotted (for the US 1987/2007 data) empirical forward rate volatility when we allow the 15-year forward rate to adjust towards a fixed long-term level along a path designed to produce this reasonable level of forward rate volatility. We have started at year 15 since forward rates at this maturity still look well behaved.

You can see that the forward rate volatility exhibited by the 'manufactured' curves is broadly in line with the targets over this period. In practice there will be some modest variation in the UFR itself but this will not have a material impact on this profile. Exhibit 6 shows the manufactured forward curves. You can see that the variation observed in 15-year forward rates decays so that, although 50-year forward rates still exhibit considerable variation, most of this is assumed to have died away at a 100-year horizon.

Finally, exhibit 7 shows the impact on discount bond volatility of extrapolating from various horizons. It can be seen that, for the USD data, an extrapolation from the 10-20 year area using our methods results in price volatility of the 100-year zero around 30-35%. We cannot say for sure whether this is right or wrong since we cannot see the way variations in the risk premia embedded into observable forward rates are transmitted to untraded, unobservable long maturities. However, we can say that these profiles are more economically appealing and may avoid generating spurious volatility in the value of the ultra long-term untraded cash flows on the balance sheet of insurance firms and pension funds.

You should take careful note of the two important features of our extrapolation approach: first, damping the apparent spurious volatility in long-term forward interest rates; second, extrapolating towards a slow-moving ultra-long-term target.

Exhibit 5

Forward rate volatility targets and US forward rate volatility extrapolated from year 15

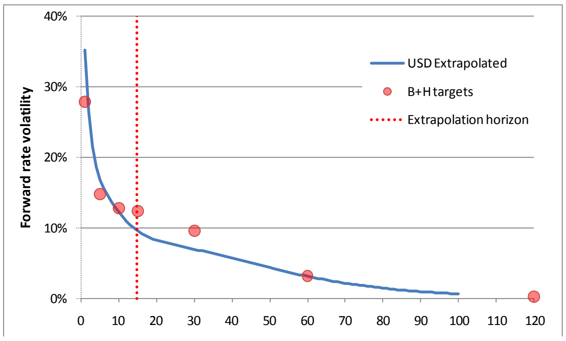


Exhibit 6

USD government forward rates extrapolated from year 15 using B+H method

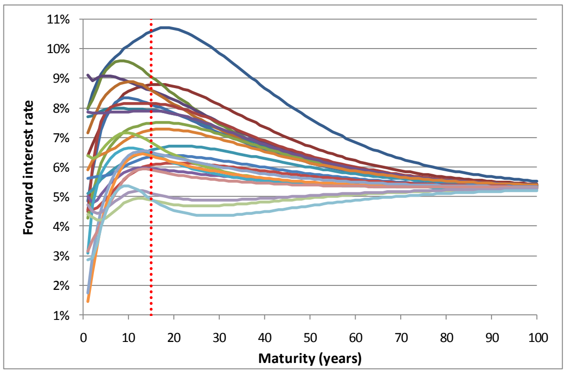


Exhibit 7

Discount bond volatility for US government forward rates extrapolated from various maturities towards a fixed unconditional forward rate in line with proposed forward rate volatility decay

Longest maturity for extrapolation (years)	*Term of discount bond (years)* 30	50	70	100
30	21%	41%	59%	70%
20	21%	28%	32%	35%
10	21%	26%	28%	29%
5	29%	42%	48%	52%

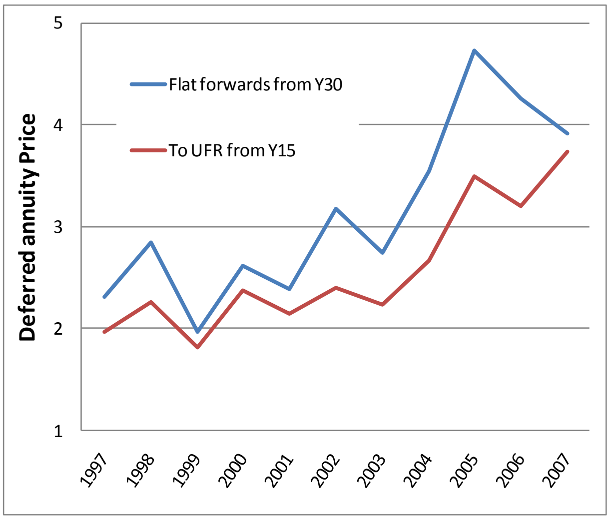
[10] *See our forthcoming calibration note: 'Real-world interest rate calibration: How to construct a volatility term-structure of interest rates in the absence of market prices'.*

An example deferred annuity liability

Let us look briefly at the impact on an example long-term liability. Suppose we use the extrapolated curves to price a 25-year annuity (paid annually in arrears) deferred for 30 years. The chart below shows how the annuity price would have varied between 1997 and 2007 using, alternatively a flat forward assumption at year 30 or an extrapolation towards a fixed unconditional forward rate from year 15 as described above. You can see that, over the 10-year period, the dollar value of the annuity liability starts and finishes at similar levels for the two approaches but that extrapolation using flat forward rates generates more variation in the estimated value of the annuity. In 2005, the peak value of the annuity using a flat extrapolated curve arises because of the very low estimated 30-year forward rate of 3%. Two years later, at end-2007, this estimated forward rate had risen above 4%. The interesting questions for us are: first, whether it is reasonable to take these long-term forward rate estimates at face value and, second, whether it is sensible to assume they persist into the far future. At the risk of repetition, we do not – we propose to smooth the terminal forward rate and to extrapolate towards a slow-moving economically justifiable target long-term forward rate assumption.

Exhibit 8

USD price of a 25-year annuity deferred for 30 years using alternative forward interest rate extrapolation methods



Summary

The path of extrapolated yield curves and the consequent volatility of ultra long-term discount bonds and known cash flows depends directly on the view taken on how variations in interest rate term premia are transmitted through the yield curve. There are two schools of thought. Firstly, those who believe (implicitly or explicitly – it doesn't matter) that risk premium changes observed at short maturities are propagated throughout the entire term structure. This extreme view generates huge volatility in ultra long-term discount bond and cash flow values. Secondly, an alternative view (with which we agree) that the variability in risk premia declines with term. This is a view motivated by economic thinking and observation of the behaviour of forward rates at shorter maturities, albeit distorted by measurement errors.

Those who choose the apparently innocuous approach of extrapolating with constant forward rates will soon discover what a strong and influential decision it is. Think hard before you do it.

More detail on our thinking regarding the extrapolation of yield curves is contained in the Exposure Draft prepared for B+H's technical Advisory Panel, *'A framework for estimating and extrapolating the term structure of interest rate'* (September 2008) and supporting technical documents:

'Calibration Methods - Fitting the yield curve: Spline interpolation and Nelson-Siegel extrapolation', September 2008.

'Calibration Methods - Interest rate calibration: How to set long-term interest rates in the absence of market prices', September 2008.

	QIS4 / CFO Forum constant longest forward rate	B+H method
1. *What is the longest market [forward] interest rate we can observe?*	Method is not specified.	Fit using a spline function but be prepared to smooth at longest maturities to avoid spurious forward rate volatility.
2. *For the purposes of extrapolation, what is the very long-term, 'unconditional' forward interest rate?*	The longest available estimated market forward rate.	A slow-moving assumption based on long-term fundamentals.
3. *What path should be set between the longest market rate and the unconditional forward rate?*	A flat fixed path.	A path designed to produce forward rate volatility decay which is consistent with observation and economic beliefs.
Comments	a) It is difficult to make a reliable estimate for the longest forward. b) All forward rate volatilities are matched to the volatility of estimated forward rate at the term of the longest market instrument.	a) 'True' volatility of unobservable extrapolated forward rates will depend on 'true' volatility of unobservable risk premia.

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