

Barrie & Hibbert's LMMPlus

Enhanced modelling solution for market-consistent liability valuation

LMMPlus – a new addition to Barrie & Hibbert's ESG 7 armoury

To enhance our modelling solution for market-consistent liability valuation, in ESG 7, Barrie & Hibbert are introducing Libor Market Model Plus (LMMPlus) for interest rates. LMMPlus will enable clients to:

- More accurately report on the market-consistent value of liabilities, for example meeting SII requirements for reporting of Technical Provisions.
- Better understand the value of embedded guarantees and the level of risks they entail during product design and pricing stages.
- Avoid lognormal blowup issues, even in extreme market conditions.

Is LMMPlus for you?

In this note you will find an overview of our new model's valuation performance, comparing it to our existing Libor Market Model – helping you to assess its suitability as part of your liability valuation solution.

LMMPlus - model features

LMMPlus builds upon Barrie & Hibbert's existing LMM. It adds the combination of two key modelling features:

- **Stochastic volatility**

The level of volatility in interest-rate markets changes in an unpredictable way, so our cutting-edge model reflects this, increasing the range of real-market phenomena that the model captures.

- **Displaced diffusion**

This feature relaxes the constraint that rates must strictly never reach zero. This constraint produces a low frequency of very low interest rates, potentially understating the value of very low-strike options. It allows us to shift the floor on rates from zero to lower values, resulting in more realistic dynamics in the regime of low rates, and acting as a 'dial' to shift between the classic Lognormal (eg LMM, Black-Karasinski) and Normal (eg Hull-White) modelling assumptions. As different modelling assumptions may be more/less appropriate in different market conditions, this dial is an invaluable modelling feature.

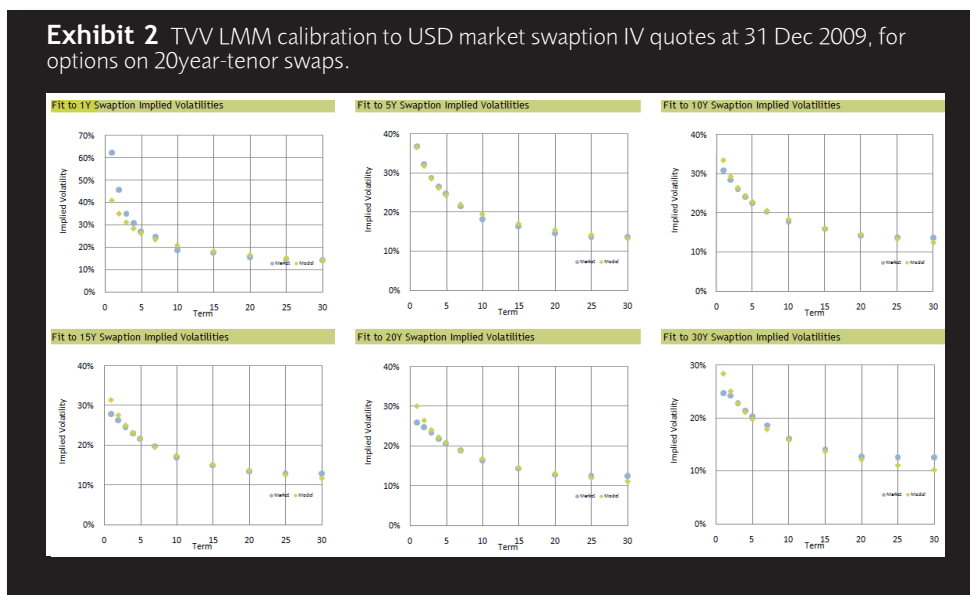
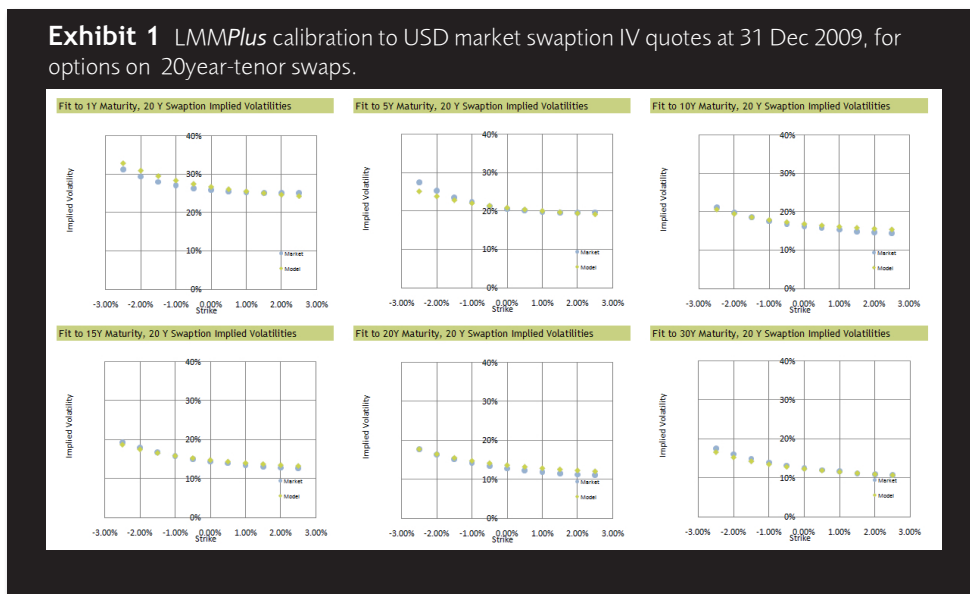
These new features result in enhanced calibration performance and more practical usability. As they are built into our existing LMM framework, using the model is straightforward for users familiar with the existing model. Achieving accurate liability valuations through the employment of these features is discussed in detail overleaf.

Capturing the shape of the option 'skew'

LMMPlus can match a wider range of market option prices than is possible with any existing suite of models. In particular, unlike the classic Lognormal LMM, LMMPlus produces variation in swaption implied-volatilities as a function of option strike. This is a well-known feature of market prices, and may have a significant impact upon the market-consistent value of embedded guarantees which are not at-the-money.

To give an indication of the model's calibration performance, we show option pricing under LMMPlus and time-varying-volatility (TV LMM), for USD swaptions at End-Dec 2009.

Each chart below shows option IV as a function of strike, for a specific option maturity¹.



ESG users will of course be interested in the impact that a change of model will have upon valuations. In a follow-up note, we will use both LMMPlus and TV LMM to value life insurance liabilities, and report on this, but for the time being, we can get an indication of the impact by looking at resulting prices of relevant instruments. In particular, Guaranteed Annuity Options (GAOs) are closely related to receiver swaptions, so it is useful to consider the difference in price under the

two models (rather than simply looking at the implied-volatility metric, which can obscure differences in raw valuation).

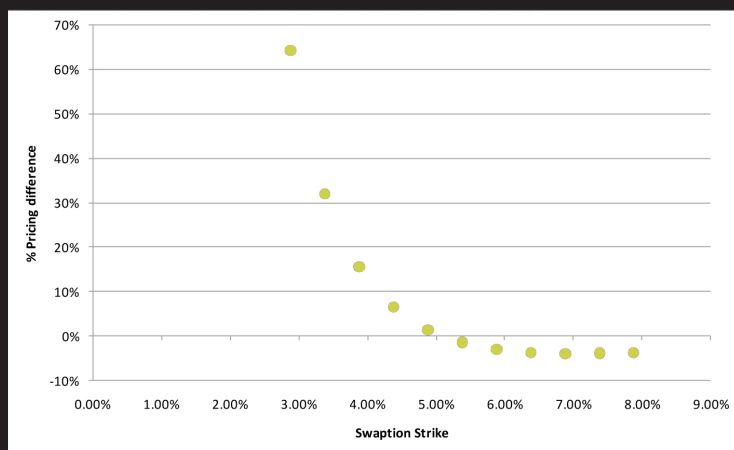
To give an illustration, we continue using the Dec 2009 USD calibration that we have looked at above – Exhibit 3 overleaf shows the difference in price between the two models, for 10 x 10 receiver swaption contract at a range of strikes.

¹The strike is referenced against the forward-ATM strike, so a strike labelled X% corresponds to an option struck at the forward-swap rate appropriate to that option, shifted by X%.

As we can see, LMMPlus captures the shape of the option 'skew', whereas a lognormal model such as TVV LMM is not capable of producing the variation of IV with strike. As a result, using LMMPlus will allow you to:

- More accurately report on the market-consistent value of liabilities, for example meeting SII requirements for reporting of Technical Provisions.
- Better understand the value of embedded guarantees, and the level of risks they entail, during product design and pricing stages, improving decision making around the profitability of new products.

Exhibit 3 Relative price difference ($P_{LMMPLUS}/P_{TVLMM} - 1$) between the two models, for 10x10 receiver swaption contracts (USD Dec 2009), as a function of swaption strike. The 'forward at-the-money' strike is a little over 5%.



In this particular case, the price of low strike options, under LMMPlus, is significantly larger than the value under the lognormal model. In contrast, options written at a high strike rate are attributed a lower value under the new model, though differences are somewhat more modest. Whilst results will be heavily dependent from one calibration to the next, this gives an indication that the value assigned to GAOs that are deep in-the-money might be expected to change significantly upon moving to a model which captures skew, such as LMMPlus.

Improved practical usability

Avoidance of lognormal blow up problems

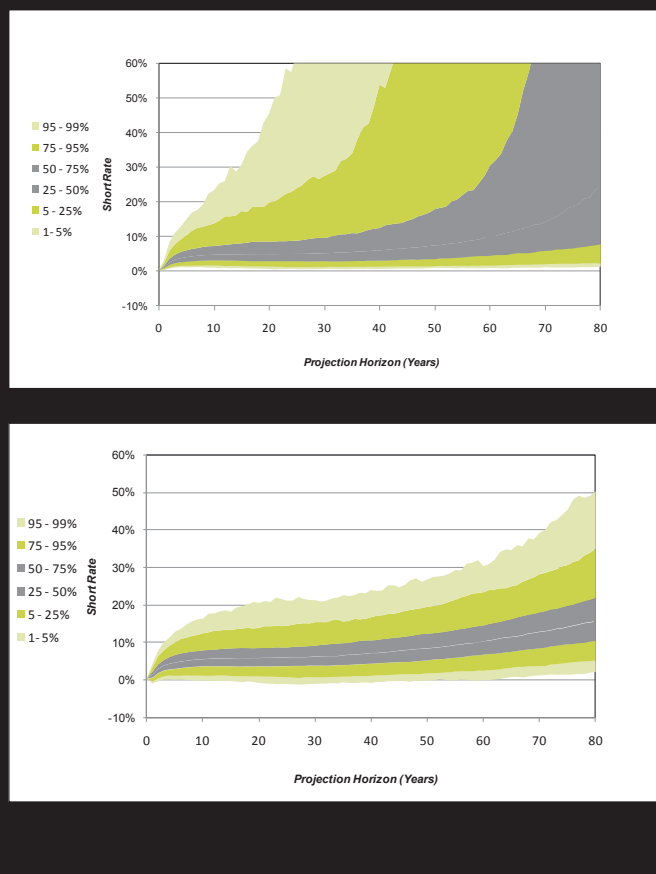
A well-known drawback of the classic Libor Market Model is 'lognormal blowup'. This term reflects the 'fat' upper tail of the lognormal distribution, which results in a relatively high probability of large interest rates, which can be problematic using Monte-Carlo techniques.

Some results of lognormal blowup, typically observed when scenarios have high volatility or a very long time horizon, are:

- Insurance liability-modelling systems may be unable to deal with extreme rates and resulting asset indexes, leading to scenarios 'skipping'.
- If any cap on rates significantly truncates the model's natural rate distribution, then scenarios may no longer pass standard scenario tests (eg, Martingale test, option IV tests).
- Sampling from the highly-skewed distribution produced by lognormal models can result in large sampling error in Monte-Carlo pricing, leading to difficulties passing standard scenario tests.

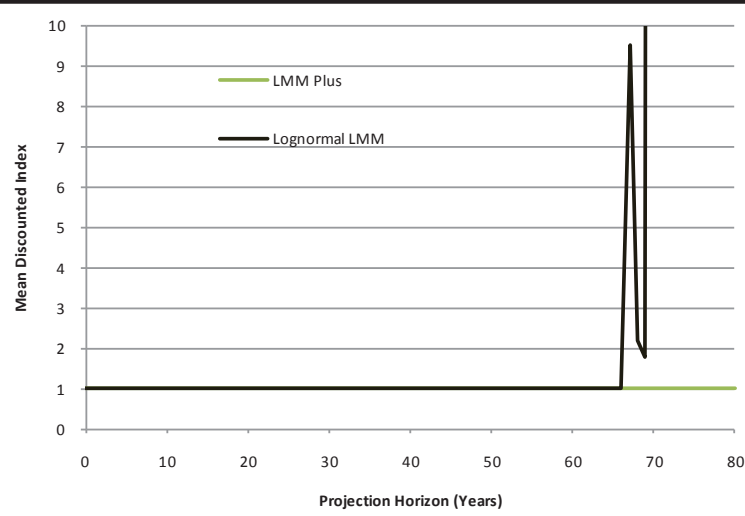
By using LMMPlus, you will be able to avoid these issues because the model allows you to alter the distribution of rates from lognormal. This results in significant reduction of the upper tails, and in the presence of some small negative interest rates. Both of these features can clearly be seen in Exhibit 4.

Exhibit 4 Comparison of short (1-month) rate under lognormal LMM (upper) and LMMPlus (lower). The LMM Plus results are produced using a displacement of approximately 2%. Monthly implementations, calibrated to USD swaptions at Dec 2009. Common scales are chosen to allow comparison.



As a result of the change in distribution, the model offers excellent Monte-Carlo performance, even over very long horizons, and even in the presence of extreme market conditions. This can be seen in the performance of Martingale tests, as shown in Exhibit 5 overleaf.

Exhibit 5 Comparison of martingale tests under LMMPlus and Lognormal LMM, for an index of 10-year default-free ZC bonds (with maturity rebalanced to 10-years each month). The forward-rate cap is set at 1000%. For lognormal LMM, the presence of the cap results in erratic performance at long projection horizons.



What is the impact of this change in the interest-rate distribution? To illustrate the impact, we can compare the behaviour of martingale tests under the two models. As mentioned above, many practitioners have used a cap to prevent the appearance of very large rates. However, over a very long horizon the lognormal LMM will naturally produce a significant number of scenarios that exceed the cap level – and when this occurs the scenarios will produce noticeable pricing errors, causing Martingale tests to fail. With the displacement chosen here (approximately 2%), LMMPlus produces a much narrower rate distribution, avoiding the problem and so producing Martingale tests which perform well even at very-long horizons.

Find out how Barrie & Hibbert can help your business manage risk more effectively

Our forthcoming series of Technical documents will provide greater technical detail of LMMPlus, together with details of our work on utilising LMMPlus for real-world projection applications. Should you require further information prior to their publication, please get in touch with your usual Barrie & Hibbert contact, or email us at: info@barrhibb.com.

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