There are a number of ways to calculate capital for solvency and regulatory purposes. Here we’ll provide a brief overview of five common and not so common methods used for calculating the solvency capital requirement (SCR):

- **Covariance Matrix**
- **Curve Fitting**
- **Least Squares Monte Carlo**
- **Replicating Portfolios**
- **Nested Stochastic**

We’ll be covering mainly the SCR for market risk but a lot of these techniques are equally applicable to capital calculation for other types of risks.

We’ll base the discussion on the definition of capital described in the Solvency II directive, using a 99.5% value-at-risk measure for net assets over one year. If you do use another method, you still have to make sure it’s as stringent as the one-year VaR method, so I can’t imagine many people straying away from this. Although this is the basic formula in the directive, it’s nevertheless a difficult task.

Ideally we would like to know what the full probability distribution of net assets looks like in one year. This is very difficult to do as it is a nested stochastic problem which, in theory, requires a full nested simulation to give an accurate solution. (We would generate thousands of simulations of the real-world evolution of the economy over 1 year and, for each of those, run thousands of market consistent simulations to calculate the value of our liabilities and net assets.) Just to run one market-consistent valuation — to find the value of our liabilities today — can take hours or days to run on many asset liability models in use in the industry.

To calculate the full distribution we’d have to run the valuation model many times under different real-world paths which would be obviously very time-consuming. Although we may settle for knowing the shape of the left tail of the distribution which is where capital is calculated – to save time on our calculation - knowing the full distribution is nice for other things. We can use it for hedging, risk and capital management, communication with shareholders and directors.
But because of the complexity of this problem we need to make some approximations and there are a few options we can choose.

**Covariance matrix**

The covariance matrix is the most popular method currently used for UK individual capital assessment (ICA) calculations.

Instead of calculating the full distribution of your losses, you can come up with an assumption for the 99.5% instantaneous shock for each individual risk driver. Then we run our market-consistent stochastic model under each of the shocks and calculate what the loss to the company would be. Now we'll make two really big assumptions:

1. We’ll assume that our economic risks follow a multivariate normal distribution (actually it’s elliptical but most people just use normal).

2. We’ll also assume that our losses are a linear function of our risks.

Once, we’ve made those assumptions, our calculations become very easy. We can come up with some correlations and feed them into a covariance matrix. Using this simple ‘root sum squares’ formula,

\[
SCR_{market} = \sqrt{\sum_{i,j} Corr_{i,j} \times SCR_i \times SCR_j}
\]

and out comes the required aggregate capital number.

So we need just five or ten stress tests, we feed them into our formula and we come out with a result - a lot less calculation than under the nested stochastic method. However, there are a few major problems with doing this type of calculation and they rest on the assumptions we are making.

It is often said by practitioners that the covariance matrix approach assumes ‘linearity’. This term is sometimes used in a confusing way. Actually two kinds of linearity are assumed – that the dependence between economic risks can be summarised by the linear Pearson correlation coefficient (as is the case in the multivariate normal distribution) and that the loss on the insurance portfolio is a linear function of the risk drivers.

It is important to separate out these two kinds of non-linearity when we analyze any kind of approximation as both are very important for a capital calculation.

Looking at the economic risks, linearity is one part of a larger problem. We will often want to use more complex assumptions than the multivariate normal distribution. For two assets this is shown in the charts below.
The chart on the left assumes that returns come from a joint normal distribution. Those on the right are generated from a model that assumes that the two assets react to a set of common risk factors, undergo periods of high and low volatility and experience jumps in returns. Both have the same standard deviation and correlation.

When we calculate capital, we are interested in the observations in the bottom left corner so ‘fat tails’ and ‘tail dependence’ are very important to us.

If the picture on the right represents our view of market behaviour then we could try to adjust the assumptions of the covariance matrix to make the left picture look more like the right one. We would increase both the standard deviation and correlation to get the dots further down into the corner. This is not a happy compromise and rarely gives an accurate answer.

The second source of non-linearity is the behaviour of the liabilities. Liabilities are non-linear under changes in many risk drivers.

The example below shows how the value of a vanilla put option reacts to changes in the value of the underlying equity and to the risk free interest rate.
The value of a put option liability increases by more than a linear model suggests - for changes in both equity and interest rates. Liabilities also have joint non-linearity. The chart below shows the joint behaviour of a put option value under changes to both variables.

If we calculate the individual non-linearity of equities and interest rates separately and assume that when both variables change, the change in value is the sum of the two individual changes we can see that we still get a bad fit for the joint behaviour of the function.

The ‘linear joint’ approximation is accurate for changes in one risk but underestimates the put option value under simultaneous falls in equities and interest rates.

Coming back to the covariance matrix, the implicit assumption is that the liabilities behave like a flat plane that intersects the liability function at certain distinct points (chosen to be the 99.5th percentile losses in individual variables). This is shown in the graph below. The assumed liability value is individually and jointly linear.

This is by its nature only correct in some places (a bit like a stopped clock) and tends to underestimate values when liabilities are non-linear as shown below.
It is possible to replicate the covariance matrix calculation with Monte Carlo simulation. The steps are:

- Generate a large set of correlated simulations that correspond to a joint normal distribution.
- For each simulation calculate the change in balance sheet assuming a linear model fitted to 99.5% stresses.
- Find the 99.5\textsuperscript{th} percentile (or other risk measure).

You should get the same aggregate capital number as you do using the covariance matrix method.

But if you were going to go down this Monte Carlo route, would you use the simplistic assumption of linear liabilities? Realising that a straight line is rarely a good approximation for anything, you would more likely fit a more complex function.

This inaccuracy is well known. The Swiss Solvency Test, which uses a covariance matrix approach similar to the Standard Formula in Solvency II, is analysed in a paper called *When the SST Standard Model Underestimates Market Risk* by Giuseppe G. Cardi and Roland Rusnak. It finds that the approach underestimates capital required significantly for life insurance companies (but does a reasonable job for general insurers).
This is fairly worrying, given that an internal models benchmarking study carried out by Oliver Wyman states that 60 per cent of CFO forum companies currently use the covariance matrix approach to aggregate capital. The more stringent supervision likely to come with Solvency II will almost certainly focus on how insurers have handled fat tails and tails dependence in their economic assumptions and the non-linearity of their liability books.

In summary, while the covariance matrix methodology is the basis of the Solvency II standard formula, it contains glaring and substantial flaws that lead to a high level of inaccuracy. This makes a bit of a mockery of the really detailed calculations that companies make in their asset-liability modelling systems that feed into the capital calculation. But more importantly, flawed capital rules can also lead to bad decisions by companies, especially where the regulatory system discourages them from making sensible investment, risk management and hedging decisions.

While some firms and regulators use a ‘big bang’ or ‘medium bang’ approach to adjust for the non-linearity of liabilities and use overly conservative economic stresses and correlations to try to compensate for the deficiencies of the joint normal model. These ‘fixes’ are generally inaccurate individually and do not work well together. There is no way to estimate how inaccurate they are and how this inaccuracy changes from one reporting period to the next.

More sophisticated models are required and a curve-fitting approach is the next step up from a covariance matrix.

The subject of curve fitting is further discussed at: http://www.barrhibb.com/research_and_insights/article/improving_capital_approximation_using_the_curve-fitting_approach
Disclaimer

Copyright 2011 Barrie & Hibbert Limited. All rights reserved. Reproduction in whole or in part is prohibited except by prior written permission of Barrie & Hibbert Limited (SC157210) registered in Scotland at 7 Exchange Crescent, Conference Square, Edinburgh EH3 8RD.

The information in this document is believed to be correct but cannot be guaranteed. All opinions and estimates included in this document constitute our judgment as of the date indicated and are subject to change without notice. Any opinions expressed do not constitute any form of advice (including legal, tax and/or investment advice).

This document is intended for information purposes only and is not intended as an offer or recommendation to buy or sell securities. The Barrie & Hibbert group excludes all liability howsoever arising (other than liability which may not be limited or excluded at law) to any party for any loss resulting from any action taken as a result of the information provided in this document. The Barrie & Hibbert group, its clients and officers may have a position or engage in transactions in any of the securities mentioned.

Barrie & Hibbert Inc. and Barrie & Hibbert Asia Limited (company number 1240846) are both wholly owned subsidiaries of Barrie & Hibbert Limited.