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1-yr VaR assessment and dynamic management actions

With implementation projects for Solvency II's Internal Model approach to Solvency Capital Requirements getting underway, research on 1-year VaR methodology for insurance business has been very active. There has been substantial work published on the joint modelling of asset risk factors and the relative merits of approaches such as copulas and structural models. Similarly, some leading-edge work has been done on how to approximate the end-year solvency distribution by using approaches such as curve fitting and Least Squares Monte Carlo to by-pass the need for a full Stochastic-on-Stochastic implementation.

An interesting feature of the above research has been its tendency to assume that firms' Internal Model approaches to 1-year VaR will be implemented by projecting forward one year in a single modelling time increment. An Internal Model that can only project in a single time-step will not be able to capture any dependency that the solvency position may have on the path that the risk factors take over the 1-year projection. Many life insurance liabilities' solvency position will have significant path-dependency. This path-dependency may arise from dynamic management actions (for example, dynamic asset allocation and bonus rates for with-profit business) or dynamic hedging strategies (such as those commonly in place for variable annuity business). If such dynamic actions are implemented more than once a year, a multi-period 1-year Internal Model will be required to make a realistic assessment of the 1- year VaR capital assessment.

In this note, an illustrative case study is used to demonstrate the materiality of this effect in the context of dynamic management actions in with-profit business.

An illustrative case study

Consider a single-premium with-profit contract with a 5-year cash maturity guarantee of £110 on a fund with a starting value of £100. The fund's initial asset allocation is 50% in equities and 50% in a matching risk-free bond. The asset allocation of the fund is re-balanced on a monthly basis to a target equity allocation according to the formula:

Equity allocation = 5 x (Fund Value – PV of Guaranteed Sum) / Fund Value

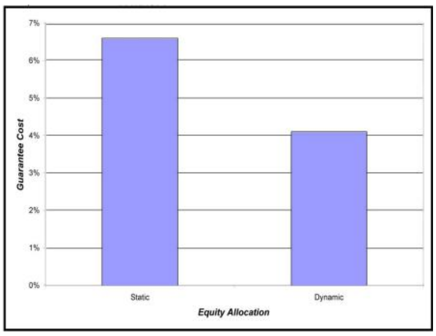
The monthly change in the equity allocation is constrained to be no greater than 2.5%, i.e. the maximum annual change in equity allocation is 30%. This is arguably an aggressively dynamic allocation rule, but this will help to illustrate the nature of the effect on capital requirements. Later in the note, we also explore the sensitivity of the capital requirement to changes in the monthly limit on the equity allocation change. (And the equity allocation is subject to a minimum of 0% and a maximum of 100%).

We can now consider the calculation of the market-consistent value of the above guarantee and its 1-year VaR requirement. In this analysis, we will assume that:

- equity total return volatility is 20% (both for market-consistent valuation and the 1-year real-world projection);
- the risk-free rate is a constant 4%;
- the real-world equity risk premium is 4% pa; and
- the equity total return is assumed to be a lognormal random walk. *(All of these above assumptions refer to continuously-compounded returns).*
- We'll also assume that charges of 1% pa are deducted from the fund value. Lapses and mortality are ignored.

Risk-neutral stochastic simulations can be used to calculate the market-consistent guarantee cost at time-0. Implementation of a stochastic simulation model of this liability with the above asset modelling assumptions produces a value of £4.1 for the market-consistent guarantee cost. Turning the dynamic equity allocation off and assuming a constant 50%/50% equity/bond mix, we find that the guarantee cost increases to £6.6.

Exhibit 1: Time 0 market-consistent guarantee cost

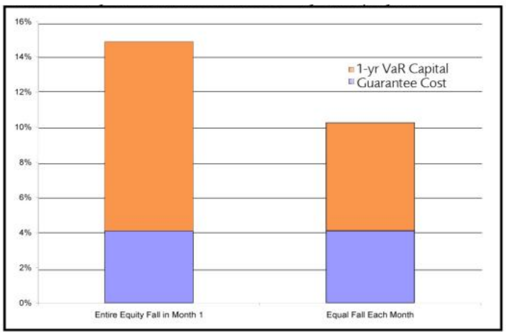


1-yr VaR assessment based on the 1-year equity model

We now turn our attention to the calculation of the 1-year VaR capital assessment. In our simple case study, the capital requirement is a function of only one risk factor: the equity total return. So a simple approach to assessing the capital requirement could be to calculate the 99.5th percentile equity fall, and calculate the guarantee cost in that scenario. The above equity modelling assumptions described above imply a 1-year 99.5th percentile equity fall of 35%. However, an immediate complication arises as a result of our monthly management actions. The presence of these actions means that the relevant risk factor is not the end-year value of the equity index, but the month-by-month path it takes over the 1-year projection horizon. We could use some simple assumptions to construct a month-by-month path from the 99.5th percentile end-year equity fall. For example:

1. The full fall is experienced in the first month, with each of the next 11 months producing an equity total return of zero.
2. The first 11 months produce zero equity total returns, and the full fall occurs in the final month of the 1-year projection.
3. An equal equity fall is experienced in each of the 12 months.

Exhibit 2: 1-year VaR based on 99.5th 1-year equity fall



Different assumptions for the 1-year return path will result in different estimates of the 99.5th percentile end-year guarantee cost, as each case implies different degrees of risk mitigation (moving assets from equities to bonds) occur prior to experiencing the equity falls. In Case 1 above, the entire equity fall occurs when the equity allocation is 50%. Whereas in Case 3, the equity allocation rule is reducing the equity allocation by 2.5% in each month, and this reduces the average amount of assets that are exposed to the assumed equity fall. As a result, in Case 1 the 99.5th percentile fund value is 82% of the starting fund value, whereas in Case 3 the 99.5th percentile fund value is 87%.

These differences have a substantial impact on the assessed 99.5th percentile end-year guarantee cost. In Case 1, the guarantee cost increases by £11.2 to £15.3 at the end of the year; in Case 3, the increase is only £6.4, and the end-year guarantee cost is £10.5.

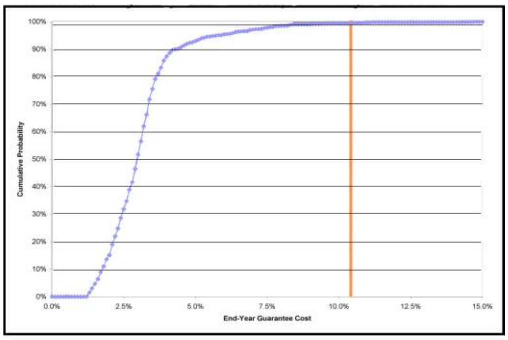
This analysis highlights how sensitive the 1-year VaR capital assessment is to the assumed path of the 1-year equity fall. A troubling aspect of the above analysis is that none of the assumed equity paths are realistic descriptions of the monthly behaviour of equity returns. So how do we know if those paths have produced a realistic allowance for the impact of monthly management actions on the required capital? We don't. A more realistic model of the month-by-month equity return path is required to make a robust assessment of the impact of the monthly asset allocation rule.

1-yr VaR assessment using a multi-period stochastic equity model

In our simple case study, we can easily develop a consistent stochastic model of monthly equity returns by running the lognormal equity model in monthly time-steps rather than a single annual time-step. The monthly model will produce the same annual 99.5th percentile fall as that used above, but this approach will also produce a realistic distribution of the month-by-month paths.

Running the equity model on a monthly basis is straightforward, but a technical challenge does arise here: we now need to value the end-year guarantee cost for every simulated end-year position of the fund, not just for the single point estimate of the 99.5th percentile as we don't have prior knowledge of which path represents the 99.5th percentile capital scenario. This creates a need for 'Stochastic-on-Stochastic'(SoS) – a full set of market-consistent scenarios is required for each simulated real-world scenario. As was mentioned above, in practice a number of techniques have been developed to approximate the full SoS implementation such as curve fitting and Least Squares Monte Carlo. But our case study is simple enough that we can implement the full SoS for our single model point.

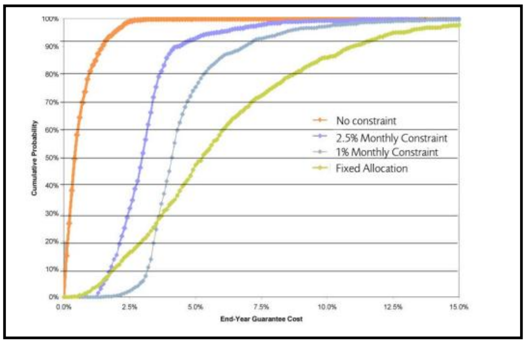
Exhibit 3: End-year guarantee cost probability distribution



The distribution of the end-year guarantee cost that is produced when monthly time-steps are used in the 1-year projection is shown to the left. The 99.5th percentile of the end-year guarantee cost is estimated at £10.4. This is substantially less than the result produced by the instantaneous stress test approach (£15.3). In this particular case, the result is very similar to the result produced by assuming the equity fall happens in 12 equal monthly increments. This won't always be the case. The stochastic monthly model is required to produce a rigorous assessment of the benefit of monthly management actions on the 1-year VaR assessment.

And, of course, once this modelling framework has been developed, it is easy to use it to investigate the impact of different degrees of management action. For example, exhibit 4 shows how the end-year guarantee cost distribution varies as the maximum monthly constraint is varied from 2.5% to 1% to 0% (i.e. fixed equity allocation), and when it is removed completely.

Exhibit 4: End year guarantee costs and management actions



So, for example, imposing a stronger constraint on the speed with which the fund can move out of equities by reducing the maximum monthly equity allocation change from 2.5% to 1% results in the 99.5th percentile end-year guarantee value increasing from £10.4 to £13.7. And moving all the way to a fixed 50% equity allocation results in a value of £18.0.

Solvency II SCR implications

CEIOPS-DOC-27/09 (formerly CP 32) describes how dynamic management actions will be permitted in the calculation of Technical Provisions providing that the modelled actions meet the CEIOPS requirements of Objectivity, Realism and Verifiability.

In the calculation of the Solvency Capital Requirement using the Standard Formula, the stresses are assumed to be instantaneous – no asset re-balancing can be assumed to occur 'through' the stress, but can be assumed to occur after the stress, again subject to the same requirements of Objectivity, Realism and Verifiability.

With the Internal Model approach to assessing the SCR, there is not the same restriction on asset allocation changes 'through' the stress. For a business that has robustly implemented and documented management action processes that occur with a frequency greater than annually, the firm can obtain credit for that produces a distribution for *realistic 1-year paths* for the relevant risk factors. Such an approach will provide a rigorous assessment of the benefits of the management actions, and is fully in the spirit of principle-based risk assessment, where the risk profile of the business should determine the modelling required to robustly measure its capital requirements.

In conclusion

This note has illustrated that for insurance business where dynamic and frequent management actions are an intrinsic part of its risk mitigation strategy, a multi-period structural modelling approach is required to robustly measure the impact of those actions on a 1-year VaR assessment.

In the context of Solvency II SCR assessment, firms with such business may have a strong incentive to use the Internal Model approach instead of the Standard Formula (which heavily restricts the credit that can be taken for the capital benefits of management actions that may be implemented during the 1-year projection horizons). It will also motivate the need for a multi-period Internal Model that can produce realistic paths for the 1-year risk factors. Stochastic paths cannot be produced by 1-year VaR methodologies such as the covariance matrix approach or a copula asset modelling approach. A structural (or causal) model of the joint paths of the economic risk factors is the most practical and effective solution for this modelling requirement.

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