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Constructing a term structure of unconditional interest rate volatility

Steffen Sorensen
steffen.sorensen@barrhibb.com

Risk managers need to make assumptions about volatility in interest rate models used for valuing assets and liabilities. An insurance company, for example, is likely to have liabilities which fall due well beyond 50 years and will need to make assumptions about uncertainty this far in the future. How do we estimate the unconditional volatility of forward 'default-free' interest rates?

When a long time-series of market data is available we can use this data to give us some guidance on an appropriate level of interest rate volatility. In practice, however, there is very limited data available which can be used to measure forward interest rate volatility at a maturity beyond, say, 10 years. Should we be lucky enough to find a long time-series of data at these maturities, there are a number of reasons why forward interest rates implied from market data may be spuriously volatile. First, spurious volatility may arise due to the method used by practitioners to construct yield curves. Second, small measurement / pricing errors of longer dated bonds can induce large variation in the long end of the forward curve. This short note summarises how we combine economic theory and available market data to construct an unconditional interest rate volatility term-structure out to 120 years.[1]

When we cannot trust the data

Let's jump straight to the problems. Figure 1 plots an estimate of conditional volatility of the change in USD nominal log instantaneous forward rates of different maturities.[2] The nominal forward rates used in the estimation are obtained from the zero-coupon yield curve constructed using the parametric method proposed by Svensson.[3] The figure illustrates how the volatility of the change in nominal forward rates decreases with maturity up to 15 years while the volatility of the 30 year forward rate is exceptionally high and fluctuates.

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[1] For full detail see Barrie & Hibbert calibration methods note: 'How to construct a volatility term-structure of interest rates in the absence of market prices', Z Liu, October 2008.

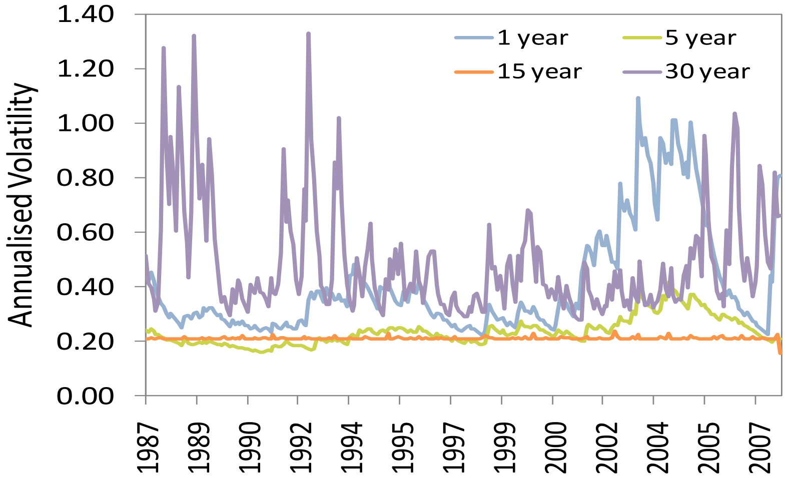
[2] Volatility estimated using simple GARCH(1,1) model.

[3] See Svensson, L. E. (1994), 'Estimating and Interpreting Forward Interest Rates: Sweden 1992-94', International Monetary Fund: Working Paper.

.........there are good arguments why the volatility term-structure of changes in nominal log forward rates must be downward sloping.

Figure 1

USD: Conditional volatility of the change in log nominal instantaneous forward rates at different maturities



Source: See Gurkaynak et al, 'The U.S. Treasury Yield Curve: 1961 to the Present', Federal Reserve Board Discussion paper 2006-28, ttp://www.federalreserve.gov/pubs/feds/2006/. Estimated using GARCH(1,1) model.

From a theoretical point of view, we would expect instantaneous nominal forward rates to tell us something about market expectations of future inflation and real interest rates. It is hard to think of a good reason why uncertainty about inflation and real interest rates should decrease over the next 15 years and then increase. Some may argue that the term-structure of interest rates contains term premia and convexity effects that may create additional volatility in longer dated forward rates. There must be some truth to this, but it is hard to believe that the presence of term premia and convexity effects can create such an effect.

Other factors are likely to be at play! Our research highlights a number of reasons why the volatility term-structure of changes in forward rates is more likely to be downward sloping:

1) Well-established theoretical term-structure models imply that the volatility of changes in the logarithm of the long-term nominal forward rate is zero as long as the short-term interest rate and term premia are stationary.[4] It is difficult, at least economically, to justify non-stationary behaviour of interest rates and term premia.

2) Yield curve construction methods used by many practitioners and central banks will introduce spurious variation in the long end of the forward curve. When using a parametric model (i.e. a Svensson type model) to construct the yield curves from available bond prices, spurious volatility is induced because the estimated terminal values are allowed to move around to give a better fit to the data.[5]

3) Small measurement/pricing errors (for long maturity financial instruments) can induce large variation in the long end of the forward rate curve. Small pricing errors have little impact on the volatility of short maturity forward rates. But their effects on the long maturity financial instrument can be large.

[4] All we need for this result to hold is that the term-structure model is (semi) affine. See, for example, Vasicek, O. (1977), 'An equilibrium characterization of the term structure', Journal of Financial Economics, 5.

[5] Caution needs to be taken when valuing longer dated liabilities using a yield curve constructed using a parametric model. Our yield curve construction methodology - a spline-based method extrapolating to an unconditional level established using economic theory – does not introduce such spurious volatility.

We believe that the volatility term-structure of changes in the forward rates must be downward sloping. Our research suggests that it is prudent to disregard current information about volatility in forward rates beyond a 15 year maturity.

When we do trust the data

We are fairly comfortable with the information on volatility in nominal forward rates of maturities between 1 and 15 years. For such maturities we have a long data series in the US (starting 1962), UK (starting 1971), Germany (starting 1973) and Canada (starting 1987) to enable us to judge a sensible assumption on volatility of interest rates. But two difficult questions remain: 1) Should we weight historical observations equally or should more weight be attached to more recent observations? 2) Should we weight interest rates equally across different economies?

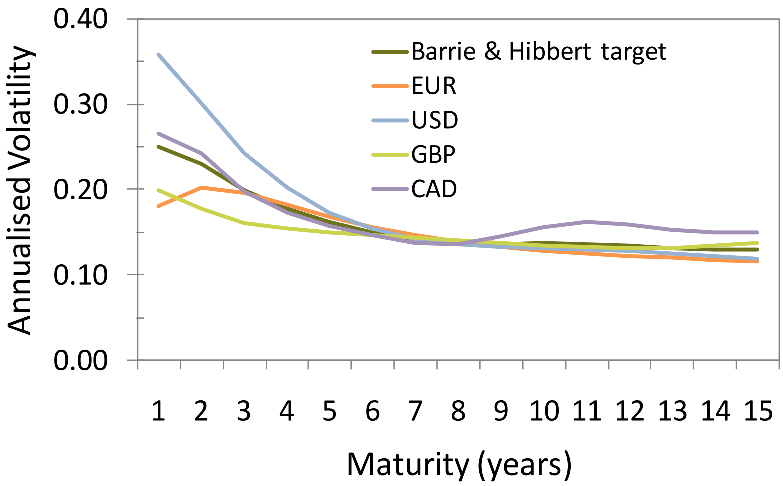
On the first point, we think it is sensible to attach more weight to recent observations as these will include information about potential structural changes in an economy. But consistent with our standard practices, we only allow new information to update our unconditional volatility estimates gradually as new information is reflected in the data. On the second point, there does appear to be evidence that short-term volatility has differed substantially across different economies in the past. At the longer end, on the other hand, we would not expect material differences across different economies. We do think, however, that it would make sense to pool the available data and set a target volatility term-structure that is equal to the average volatility across the four economies included in our analysis. We think this is sensible as it is not necessarily the case that the past is representative of the future.

.........while short-term interest rate volatility may have differed across economies in the past, we think that there is little difference at maturities beyond 5 years

Figure 2 shows our estimates of unconditional volatility constructed from available market data out to a maturity of 15 years.

Figure 2

Target for the unconditional volatility term-structure of changes in nominal log forward interest rates with maturities ranging from 1 to15 years



Extrapolating beyond the observable

The final part of our exercise extrapolates the volatility term-structure from 15 years to maturities between 50 and 100 years (typically the longest maturity of the liabilities of insurance companies). Once again it is useful to use economic theory to give us some guidance.

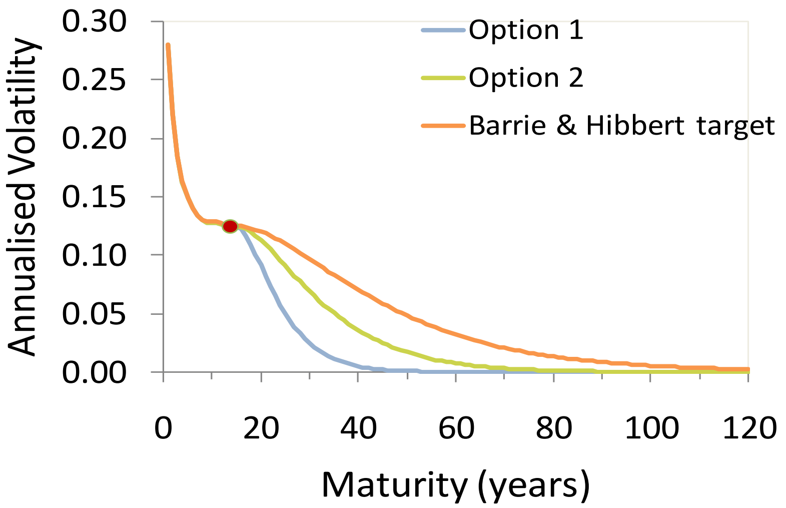
First, as explained above, it is possible to show that the volatility of changes in the log forward rates must be zero at an infinite maturity. The only criteria necessary for this to hold is an assumption that policy interest rates and term premia are stationary. Second, it is possible to show that many finance/economic models of the term-structure of interest rates imply a relationship between forward rate volatility and maturity which is very similar to the Nelson-Siegel functional form. Obviously, as shown in figure 3, there are a number of options for choosing the parameters in the Nelson-Siegel functional form to extrapolate from the last available market data point. Option

You can obtain the entire unconditional volatility term-structure if you combine economic theory with the signals you extract from available market data!

1 and 2 in the figure show two possible shapes for the volatility term-structure beyond available market data where the parameters used for the extrapolation are chosen to get a relatively fast decay in the volatility term-structure.

Figure 3

The volatility term-structure of the changes in log nominal forward rates out to 120 years



The choice of the speed of decay is hard. Fortunately we can use our target volatility term-structure of forward rates between a 1 and 15 years maturity to select parameters for the extrapolation beyond available market data out to very long maturities. Using this methodology, figure 3 shows our target for a volatility term-structure out to a maturity of 120 years. The figure shows that care needs to be taken when choosing parameters as arbitrary assumptions about the speed of decay can have important consequences for the target volatility at longer horizons.

Risk management is challenging! Available market price and extrapolation can be used to give some guidance for the choice of parameters you set models to project future risk. But setting such parameters requires a commonsense check of available market prices, guidance from economics to interpret the signals from market prices and judge sensible methods to extrapolate beyond available market prices.

Contact Us

Head Office
Barrie & Hibbert Ltd
7 Exchange Crescent
Conference Square
Edinburgh
EH3 8RD

Tel: 0131 625 0203
Fax: 0131 625 0215

Please contact info@barrhibb.com

North America Enquiries
Barrie & Hibbert Inc.
40 Wall Street, 28th Floor
New York,
NY 10005

Tel: 646-512-5750
Fax: 646-512-5756

Please contact us@barrhibb.com

EMEA Enquiries
Barrie & Hibbert Ltd
41 Lothbury
London
EC2R 7HG

Tel: 0203 170 6145
Fax: 0131 625 0215

Please contact emea@barrhibb.com

Asia Pacific Enquiries
Barrie & Hibbert Asia Ltd
Level 39, One Exchange Square
8 Connaught Place
Central Hong Kong

Tel: +852-3101 7561
Fax: +852-3101 7530

Please contact asiapac@barrhibb.com

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